Wear 368-369 (2016) 358–367

Contents lists available at ScienceDirect

Wear

journal homepage: www.elsevier.com/locate/wear

Identification of contact parameters from elastic-plastic impact of hard sphere and elastic half space

Hany A. Sherif*, Fahad A. Almufadi

Department of Mechanical Eng., College of Engineering, Qassim University, Saudi Arabia

ARTICLE INFO

Article history:
Received 26 May 2016
Received in revised form
8 October 2016
Accepted 13 October 2016
Available online 15 October 2016

Keywords:
Contact mechanics
Hertzian contact
Elastic-plastic contact
Impact

ABSTRACT

The theoretical and experimental analysis described in this paper demonstrate a simple systematic procedure to determine the contact parameters between a spherical impactor and an elastic half space via impact tests. Two main contact parameters were measured at different velocities of impact, that is the time duration of contact and the maximum normal contact force. A general nonlinear contact model based on power law is used to extract the contact stiffness and frequency of contact resonance from these two main contact parameters. Experimental results were obtained and compared for three steel samples with different hardness numbers and impacted by different sphere diameters of 2.0, 2.5 and 3.5 mm to investigate the limits of elastic deformation. Effect of the used sphere diameter on the contact parameters are shown. The test results show that the adopted model using independently measured material data can predict the contact parameters due to hard sphere impact with elastic half space. The general trend shows the increase of contact stiffness with the sphere diameter but the limits of elastic regime decrease.

1. Introduction

Impact of hard steel on massive elastic half space has long been used to study the dynamics of elastic and plastic contact [1–5]. Hertz theory was the backbone of most dynamic elastic impact models. The main difference between static and dynamic contact problems is the time duration of contact. Hertz theory [6,7] is based on assumption of fully elastic impact while most impacts are actually not fully elastic. Considering elastic and plastic contact models are more complicated than the contact law presented by Hertz [8,9]. Johnson [10] has shown that the response of an elastic half-space to dynamic impact of a rigid body to its surface can be modeled as a mass-spring –dashpot system. Much work discussed the relationship between the contact stiffness and the contact coefficients defined by Hertzian contact laws via acoustic emission [11,12] or vibration analysis [13] due to impact test. Contact parameters such as surface displacement, contact time and impact force due to collisions of steel spheres with an aluminum thick plate at moderate speed have been theoretically calculated via an elastoplastic model and experimentally verified using acoustic emission technique [12]. Elastoplastic contact models have been presented through analytical and experimental analysis of impact between a solid striker and steel or composite half-space [14–17]. Xiao et al. [18] presented a general contact stiffness model which has a force deflection characteristic with an arbitrary rational positive power to study the free vibration of an elastic sphere on a flat rigid surface.

In this paper, an elastic-plastic model is presented via a general nonlinear contact model. This model shows that the three most relevant parameters for all models of impact (for $n>1$) are the speed of impacts v_o, the contact time t_c, and the maximum normal force of contact F^*. A simple procedure for using measured data of time duration-impact velocity relationship to determine contact parameters is presented. A very light piezoelectric force transducer is used to measure the normal contact force induced by steel spheres having different diameters and striking steel plates at different impact velocities. It is shown that contact parameters like contact stiffness, elastic normal approach, contact frequency and contact size at yielding can be obtained using the measurements of the temporal force signals at different speeds of impacts. Effects of sphere diameter, and material hardness on yielding conditions of the plate material are reported.

2. Elastic-plastic impact model

Consider a rigid sphere of mass m and diameter $d=2R$ whose contact with an elastic half-space is idealized by a dashpot and spring as shown in Fig. 1. The distance y (the approach) which represents the maximum relative compression between the

* Corresponding author.
E-mail address: hasherif@qec.edu.sa (H.A. Sherif).

http://dx.doi.org/10.1016/j.wear.2016.10.006

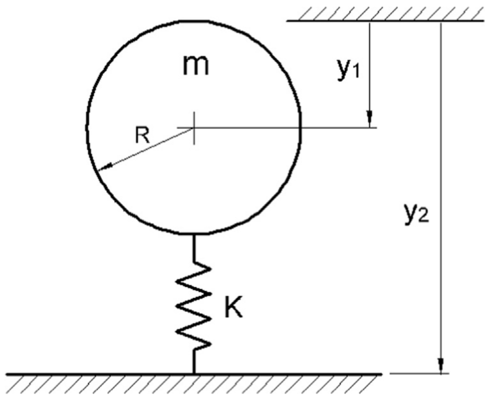

Fig. 1. Contact model of impact.

sphere and half-space can be expressed as:

$$y = (y_1 + R) - y_2, \ \ \textit{and} \ \ddot{y} = \ddot{y}_1 - \ddot{y}_2 \tag{1}$$

Assuming nonlinear elastic force,

$$F = ky^n \tag{2}$$

It is useful to mention that the positive exponent n of the elastic force in Eq. (2) has different values related to the considered physical systems. For instance, $n=1.5$ for a rigid sphere in contact with a smooth elastic half-space [8], $n=2$ for an elastic conical indenter in contact with a rigid flat surface [19], while it ranges from 2.2 to 3.5 for pianos hammers [20].

Applying Newton's second low gives:

$$m\ddot{y}(t) + ky^n(t) = 0 \tag{3}$$

where k is a constant which depends on the geometry and material properties of both

the sphere and the half-space and it has the unit ($\mathrm{N\,m^{-n}}$).

The elastic deformation $y(t)$ can be obtained by first direct integration of Eq. (3) as:

$$\dot{y}\frac{d\dot{y}}{dy} = -\frac{k}{m}y^n$$

Let $\lambda = \sqrt{\frac{k}{m}}$ be a system contact coefficient

$$\int_{v_0}^{\dot{y}} \dot{y}\,d\dot{y} = -\lambda^2 \int_0^y y^n dy$$

$$\frac{1}{2}\left(v_0^2 - \dot{y}^2\right) = \frac{\lambda^2}{n+1}y^{n+1} \tag{4}$$

If t^* is the time of maximum compression and y^* is the maximum displacement, therefore at $t=t^*$, $y=y^*$ and $dy/dt=0$, y^* can be obtained from Eq. (4) as:

$$y^* = \left(\frac{n+1}{2}\frac{v_0^2}{\lambda^2}\right)^{1/n+1} \tag{5}$$

Eq. (5) shows that the maximum displacement $y*$ is a function of velocity of impact. The second indirect integration of Eq. (4) leads to the time t^*, where:

$$\int_0^{t^*} dt = \frac{1}{v_0}\int_0^{y^*} \frac{dy}{\sqrt{1-\left(\frac{y}{y^*}\right)^{n+1}}}$$

The solution is given (Appendix) as:

$$t^* = \frac{t_c}{2} = \frac{y^*}{v_0}\int_0^1 \frac{dy}{\sqrt{1-x^{n+1}}} = \frac{y^*}{v_0}\left(\frac{\sqrt{\pi}}{n+1}\right)\frac{\Gamma\left(\frac{1}{n+1}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{n+1}\right)} \tag{6}$$

2.1. Contact time – velocity dependence

The relationship between the t_c and v_0 can be obtained by eliminating y^* between Eqs. (5) and (6); i.e.,

$$t_c = \left(\frac{2}{n+1}\right)^{n/n+1}\left(\frac{1}{\lambda^2}\right)^{1/n+1}\left(\frac{1}{v_0}\right)^{n-1/n+1}\frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{1+n}\right)}{\Gamma\left(\frac{1}{2}+\frac{1}{n+1}\right)} \tag{7}$$

It is useful to mention that at $n=1$ (linear model), the time of contact is independent of the velocity of impact and is simply given as:

$$t_c = \frac{\pi}{\lambda}$$

Only in this case, λ can be defined as the angular frequency of contact resonance. Moreover, for $n=1.5$, and $\lambda^2 = \frac{4}{3}\frac{E^*\sqrt{R}}{m}$ (Hertz model) with sphere mass $m = \frac{4}{3}\pi R^3\rho$, ρ is sphere density, effective modulus of elasticity $E^* = \frac{E}{2(1-\nu^2)}$ and ν is Poisson's ratio, Eq. (7) has the form:

$$t_c = 3.219\left(\frac{1}{\lambda^4 v_0}\right)^{1/5} = 2.869\left(\frac{m^2}{E^{*2}R\,v_0}\right)^{1/5} = 5.087\left(\frac{\rho^2 R^5}{E^{*2}v_0}\right)^{1/5} \tag{8}$$

Thus, for general nonlinear model ($n>1$) the time of contact is velocity dependent. Fig. 2 shows the variation of the contact time with the velocity of impact at different values of the exponent n from Eq. (7).

It is shown that the time duration of contact increases with power exponent n irrespective of the value of impact velocity.

Using also Eq. (7), the effect of contact constant k on the contact time-velocity relationship is shown in Fig. 3 at constant mass m and exponent n. It is shown that the contact time t_c decreases with increase of k.

2.2. Contact stiffness and force displacement relationship

At maximum displacement y^*, Eq. (2) has the form:

$$F^* = ky^{*n} \tag{9}$$

where F^* is the maximum normal force of contact.

Differentiation of F^* in Eq. (9) w.r.t. y^* results in contact stiffness as:

$$S = \frac{\partial F^*}{\partial y^*} = \mathrm{nm}\ \lambda^2 y^{*(n-1)} \tag{10}$$

Fig. 4 shows the variation of the normal contact stiffness with normal force at different values of the exponent n from Eq. (10). It is shown that at a constant normal force of contact, the contact

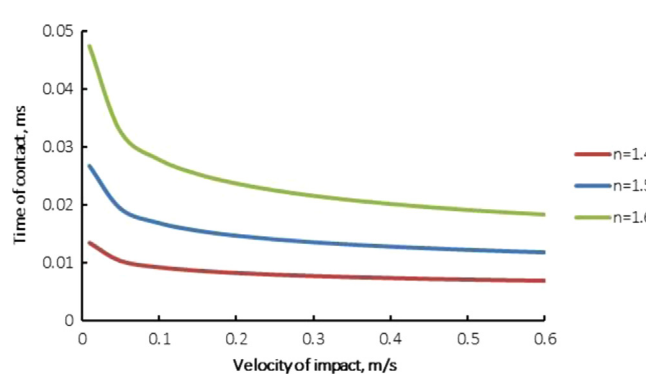

Fig. 2. Effect of exponent n on the contact time-velocity relationship ($m=0.1$ g, $k=5$ GN/m^n).