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Semi-analytical and numerical analysis of sliding asperity interaction for power-law hardening materials

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ABSTRACT

The study of the sliding process between asperities on rough surfaces can improve the understanding of wear mechanisms. The sliding interaction between asperities is analyzed in this paper using both a semi-analytical model and a finite element model. Power-law hardening materials are considered, and the asperity profiles are assumed to be a parabolic approximation to the cylinder. The effects of strain hardening exponents on some contact parameters are explored with the finite element model. Results show that the faster semi-analytical model agrees well with the finite element model for materials with larger hardening exponents, while for materials with smaller exponents, the errors would preclude its use. As the exponent decreases, the dragging effect in sliding becomes more notable and influences the contact parameters more significantly. Friction shows a significant effect in the sliding process after preliminary consideration, which should be explored in detail further.

1. Introduction

Sliding is present in many engineering applications where there are moving surfaces and plays an important role in mechanical behavior, such as friction and wear [1–9]. As is well-known, engineering surfaces are rough at the microscopic scale, containing distributions of asperities. When relative motion occurs between two surfaces, the asperities will interact with each other, and a fundamental issue is the mechanical effects of sliding asperity interaction.

Previous research in the literature has focused on asperity interaction. Normal contact is reviewed first. Hertz [10] considered contact between elastic, frictionless solids. Later, Greenwood and Williamson [11] advanced Hertz theory and gave a statistical description of surface parameters to study the normal contact between rough surfaces; research by others followed [12,13]. Kogut and Etsion [14] developed a finite element (FE) model to study the elastic–plastic contact between a rigid flat and a hemisphere under frictionless condition and gave dimensionless expressions for the contact parameters. Jackson and Green [15] improved upon the FE model with finer meshes, and explored the effects of geometry and material on hardness. These two FE models focused on the loading process. Etsion et al. [16] developed another FE model to consider also the unloading process of an elastic–plastic loaded spherical contact and gave equations for the residual interference after complete unloading. Later, Zhao et al. [17] studied the frictionless contact of a power-law hardening elastic–plastic sphere with a rigid flat. They explored the effects of the strain hardening exponents on the contact parameters during the loading and unloading processes. The residual interferences after complete unloading at different strain hardening exponents were also given.

In addition to the above models concentrating on the normal contact between asperities, other investigations studied sliding asperity interaction. Hamilton and Goodman [18] studied circular sliding contact and gave analytical expressions and graphs of the yield parameter and tensile stress distribution. Hamilton [19] then presented improved equations for the stresses beneath a sliding, normally loaded Hertzian contact. Tangena and Wijnhoven [20] first gave a two-dimensional (2D) FE model to describe the interaction between an elastic–plastic asperity and a rigid asperity, which moved through the soft asperity. Faulkner and Arnell [21] developed the first three-dimensional (3D) FE model to simulate the sliding interaction between elastoplastic hemispherical asperities. With this model, they obtained the normal and shear forces during the sliding. Vijaywargiya and Green [22] gave a thorough investigation of the forces, deformations, stress and energy loss during the sliding process between two elastic–plastic cylinders with the FE method. Jackson et al. [23] used a more rapid semi-analytical method to study the line-hardening elastic–plastic

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Nomenclature

A	Contact area
A_c	Critical contact area of asperity summit
$A_c(r)$	Critical contact area at the contact point
A_{Hertz}	Hertzian contact area
C	Critical yield stress coefficient
E_1, E_2	Young's moduli of two asperities 1 and 2
E'	Combined Young's moduli
F	Asperity contact force
F_1, F_2	Contact force of asperity 1 and 2
F_c	Critical contact force of asperity summit
$F_c(r)$	Critical contact force at the contact point
F_{Hertz}	Hertzian contact force
F_n	Normal component of the contact force
F_t	Tangential component of the contact force
i	A arbitrary sliding step number
j	Total number of sliding step
k	Empirical factor in semi-analytical model
n	Hardening exponent
R	Radius of summit of asperity
R_1, R_2	Radii of summits of asperities 1 and 2
R_s	Sum of asperity summit radii
$R(r)_1$, $R(r)_2$	Radii at the contact point of asperities 1 and 2
$R_v(r)$	Equivalent radius at the contact point
r	Horizontal position of the upper asperity
S_y	Yield strength
U	Energy loss in the sliding process
U_c	Critical elastic energy
w	Interference
w_1, w_2	Interference of asperities 1 and 2
w_c	Critical interference of asperity summit
$w_c(r)$	Critical interference at the contact point
w_{res}	Residual interference
w_y	Interference defined by Greenwood and Tripp [12]
y	Maximum vertical deformation on the profile of asperity 1
α	Contact angle between two asperities
δ	Overlap of the two asperities
ν_1, ν_2	Poisson ratio of asperities 1 and 2

asperity sliding process. They treated sliding as a process having many loading and unloading stages by using the empirical expressions of contact parameters given in the above normal contact models [12–14]. Additionally, they developed a FE model to formulate empirical expressions for the tangential and normal forces in sliding interaction, and compared them with the semi-analytical model. Results showed that these two models could match well for some but not all cases. Mulvihill et al. [24] developed a FE model for the interaction of an elastic–plastic asperity junction based on cylindrical or spherical asperities. They considered large overlaps, interface shear strength and material failure and derived a means for the prediction of friction coefficients. Dawkins and Neu [25] developed a crystal plasticity finite element model to consider the influence of the crystal orientation in the sliding process. It suggested that the plastic strain and stress fields obtained by crystal plasticity are considerably different with those given by conventional isotropic *J*2 plasticity. This is an interesting study, and the crystal plasticity will be considered in the future. However, in this work, the continuum plasticity will still be used like some previous works [22,23]. Fleck et al. [26] gave the strain gradient theory of rate independent plasticity, however, many similar finite element models developed in [14–17] considered the metal materials (e.g. steel, copper etc.) without the strain-gradient approach, where the grain sizes of the materials were just like those in this work. Those models were verified by the in-situ and real-time optical experimental investigations [27,28]. Therefore, it might not be necessary to consider the strain-gradient approach at the current scale, and for the current grain sizes, though the strain-gradient approach might be a useful method.

From the literature survey, it is clear that considerable research about sliding asperity interactions has been conducted. However, in most of the models, the elastic–plastic materials of the asperities were assumed as linear hardening materials with a tangent modulus of about 2% of the Young's modulus, and therefore the power-law hardening materials requires additional investigation. In addition, the shapes of the asperities were usually treated as a sphere in 3D or a cylinder in 2D. While as suggested by [29,30], the parabola might be a more realistic profile of the asperity in 2D at least in some cases. It is because (1) the parabola could at least reflect the characteristic that the radii at the contact point could not be identical in the contact process, which might be a slight improvement; and (2) the asperity profile could be fitted with different kinds of parabola, which might be more feasible to catch the realistic profiles. Therefore, the sliding process between the asperities having parabolic profiles will be studied for power-law hardening materials, using a semi-analytical method and a FE model. The effect of strain hardening exponents on the contact parameters will be explored, including the vertical deformation, stress, contact area, contact forces and the energy loss in the sliding process. The deformation of the nodes on the surface can reveal how the asperity deforms with the tangential forces, and the stress contour can reveal the stress distribution and evolution, which can predict the appearance of wear. Also, the single tangential loading process in this work is the foundation of the reciprocating sliding related to wear due to the fatigue, or the sliding process for two surfaces. Thus this work might give some useful results for the wear community especially for someone who focuses on the power-law hardening materials.

The assumptions used here are as follows:

(1) Sliding is assumed to be frictionless. The frictionless condition omits the friction which really exists between the asperities, and thus only the effect of plasticity on the sliding are considered. This assumption might be not realistic, however, it can isolate the effect of plasticity. In addition, some friction cases are also considered with the FE model, and are compared with the cases under frictionless condition to explore the effect of the friction.
(2) The effect of the deformation of the bulk on regions close to the contact is not considered;
(3) Sliding is simulated as a quasi-static process;
(4) Temperature effects on the sliding process are ignored.

2. Semi-analytical model

From results of previous research under frictionless condition [15,16,31,32], Jackson et al. [23] developed a semi-analytical model for spherical asperity interaction, regarding the sliding process as a many loading-unloading process. As suggested in [23], the simplified unloading-loading process could simulate the sliding process to some extent, though the actual process is more complex. The semi-analytical model here follows their method with some improvement. Power-law hardening materials are considered, and