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Postbuckling analysis of functionally graded plates subject to compressive and thermal loads

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ABSTRACT

Postbuckling analysis of functionally graded ceramic–metal plates under edge compression and temperature field conditions is presented using the element-free *kp*-Ritz method. The first-order shear deformation plate theory is employed to account for the transverse shear strains, and the von Kármán-type nonlinear strain–displacement relationship is adopted. The effective material properties of the functionally graded plates are assumed to vary through their thickness direction according to the power-law distribution of the volume fractions of the constituents. The displacement fields are approximated in terms of a set of mesh-free kernel particle functions. Bending stiffness is estimated using a stabilised conforming nodal integration approach, and, to eliminate the membrane and shear locking effects for thin plates, the shear and membrane terms are evaluated using a direct nodal integration technique. The solutions are obtained using the arc–length iterative algorithm in combination with the modified Newton–Raphson method. The effects of the volume fraction exponent, boundary conditions and temperature distribution on postbuckling behaviour are examined.

1. Introduction

The application of functionally graded materials (FGMs) in various engineering fields has been on the increase due to their special material properties, which change gradually and smoothly through certain dimensions according to a predetermined formula. FGMs are heterogeneous and typically made of metal and ceramic. Their unique features offer great potential for use in practical engineering applications, and have inspired considerable researcher interest. Numerous studies, involving thermal stress, vibration, buckling, static and dynamic analyses, have been carried out to date.

Praveen and Reddy [1], for example, carried out transient thermoelastic analysis on functionally graded ceramic–metal plates subject to thermal and mechanical loads using a plate finite element method that accounts for the transverse shear strains, rotary inertia and moderately large rotations. Noda [2] discussed the thermal stress problems of FGMs, including the thermal stresses in a FGM plate, the thermal stress intensity factor of a FGM plate with a crack and transient thermoelasticity.

Yang and Shen [3] investigated the free vibration and parametric resonance of shear deformable functionally graded plates (FGPs) in a thermal environment, and He et al. [4] conducted shape and vibration control analysis on FGM plates with piezoelectric sensors and actuators based on a finite element formulation. Liew et al. [5] investigated the static and dynamic piezothermoelastic controls of FGM plates embedded with integrated piezoelectric sensors and actuators in a thermal gradient environment. Efraim and Eisenberger [6] performed the vibration analysis of thick annular FGM plates of variable thickness. Reddy [7] developed a theoretical formulation and finite element model for a FGM plate using the third-order shear deformation theory, whereas Vel and Batra [8] reported a three-dimensional exact solution to the vibrations of FG rectangular plates.

In addition to the aforementioned studies, researchers have also conducted various buckling and postbuckling analyses. For example, Najafizadeh and Heydari [9] investigated the thermal buckling of functionally graded circular plates based on the higher-order shear deformation plate theory, whereas Liew et al. [10] turned their attention to the postbuckling of FGM plates subject to thermo-electro-mechanical loads. Shen [11] presented thermal postbuckling analysis of FGM plates with temperature-dependent properties, and Woo et al. [12] developed an analytical solution to the postbuckling behaviour of moderately thick FGM plates. Using a 3-D finite element method, Na and Kim [13–15] conducted three dimensional thermomechanical buckling and thermal postbuckling analysis of FGM plates.

The research methods commonly adopted in the postbuckling analysis of structures include analytical [16–18], semi-analytical [19] and finite element methods [20–22]. As analytical methods are

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suitable only for simple problems, and the solutions obtained from finite element methods depend strongly on the quality of the meshes, mesh-free methods, in which the interpolations are based entirely on discrete nodes rather than meshes, have recently been developed and are now often adopted as an alternative approach in engineering analysis. The element-free Galerkin method, for example, has been employed to study static and dynamic fractures [23], thin plates [24] and thin shells [25]. The meshless local Petrov–Galerkin method has been applied to thin beam analysis [26], and the reproducing kernel particle method has been employed in numerical simulations of the large deformations of thin shell structures [27] and in metal-forming analysis [28]. The spline strip kernel particle method has been used to analyse folded plate structures [29].

In this paper, the postbuckling behaviour of FGM plates under edge compression and temperature field conditions are investigated using the element-free *kp*-Ritz method [30–32]. The first-order shear deformation plate theory and the Von Kármán assumption are employed to account for transverse shear strains, rotary inertia and moderate rotations, and the material properties are assumed to vary continuously and smoothly through the thickness according to the power-law distribution of the volume fractions of the constituents. The temperature field is considered to vary through the thickness direction alone and to be held constant in any plane. The bending stiffness of the plates is assessed using a stabilised conforming (SC) nodal integration approach [33], and the shear and membrane terms are obtained employing a direct nodal integration method. Compared with Gauss integration, the proposed integration scheme improves computational efficiency and avoids shear locking for very thin plates. To solve the nonlinear system equations, the arc–length method [34] is employed in combination with the modified Newton-Raphson technique to track the complete postbuckling path. The current formulation is verified by comparisons between the present results and those given in the literature, and the influence of the volume fraction exponent, side-to-thickness ratio, temperature field and boundary conditions on the postbuckling behaviour of FGM plates is discussed.

2. Functionally graded plates

Fig. 1 shows a ceramic–metal FGP measuring a, b and h (length, width and thickness). A coordinate system (x, y, z) is established on the middle plane of this plate. The material properties are assumed to vary through the thickness according to the following power law distribution.

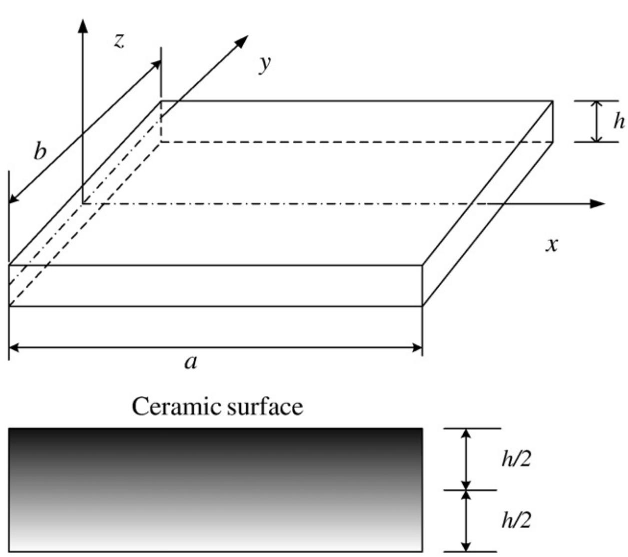

Fig. 1. A functionally graded plate.

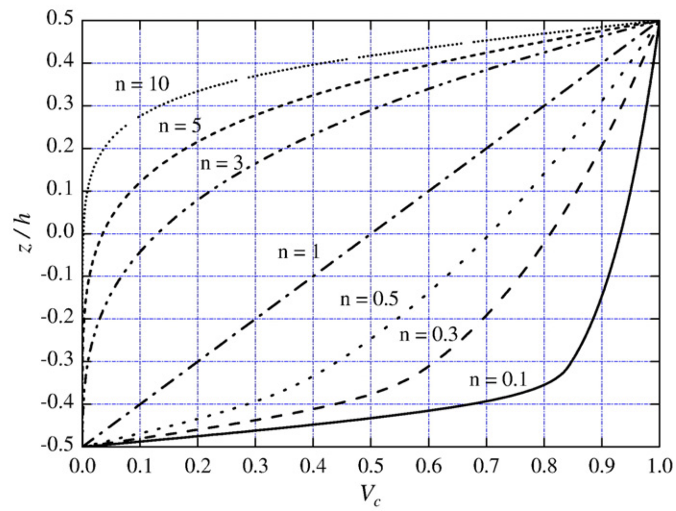

Fig. 2. The volume fraction versus the thickness.

$$P(z) = (P_c - P_m)V_c + P_m, \tag{1a}$$

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^n (n \geq 0), \tag{1b}$$

where P represents the effective material properties, including Young's modulus E, density ρ, Poisson's ratio v, thermal conductivity k and thermal expansion α; P_c and P_m denote the properties of the ceramic and the metal, respectively; V_c is the volume fraction of the ceramic; and n is the volume fraction exponent. Fig. 2 describes the variation in the volume fraction through the thickness for different volume fraction exponents n. For temperature-dependent materials, their properties are given by

$$P = P_0\left(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3\right), \tag{2}$$

where P_0, P_{-1}, P_1, P_2 and P_3 are the temperature coefficients. In this study, the material properties are computed at room temperature ($T = 27$ °C) unless otherwise specified.

3. Theoretical formulation

3.1. Energy functional

According to the first-order shear deformation plate theory [35], the displacement field can be written as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\phi_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\phi_y(x,y) \\ w(x,y,z) &= w_0(x,y), \end{aligned} \tag{3}$$

where u_0, v_0 and w_0 denote the displacements at the mid-plane of the plate in the x, y and z directions, and ϕ_x and ϕ_y represent the transverse normal rotations about the y and x axes, respectively. The nonlinear strain–displacement relationship is given by

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa}, \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \boldsymbol{\gamma}_0, \tag{4a, b}$$