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Computationally determined just suspended speed using multiphase mean age theory

David C. Russ, R. Eric Berson *

University of Louisville, Department of Chemical Engineering, Louisville, KY 40292, USA

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ABSTRACT

Mean age theory was applied towards predicting just suspended speed in mixing tanks by evaluating multiphase mean age near the bottom surface through strategic zone selection. Multiphase mean age equations were solved only in a thin section along the bottom of the vessel (~1% of the vessel height), allowing the mean age in proximity to the bottom to be computed. A rigorously defined method for open systems and a modified method for closed systems using modified boundaries provided equivalent results. The technique was accurate within 1–3% of experimental values across a range of solid densities, solid fractions, and particle sizes while using multiple impeller types and vessel geometries.

1. Introduction

Inefficient mixing, which leads to inefficient reactions, excessive power draw (Russ et al., 2014), etc., is estimated to cost tens of billions of dollars every year (Paul et al., 2004). Poor solid–liquid mixing is a culprit in mixing inefficiencies. Knowing the relative behavior of independent phases is crucial to improving performance. In most solid–liquid systems, suspension of the solid particle is essential to maintaining adequate mass transfer. The particle resting on the bottom of the tank does not have its entire surface area exposed, and thus transfer of mass between the solid and liquid phases is inhibited. Much research has focused on determining just suspended speed (N_{JS}), the minimum impeller speed necessary for all solid particles to maintain off-bottom suspension. Zwietering (1958) first put forth the concept as the minimum impeller speed required to ensure no particle remained on the bottom of the tank for more than 1–2 s, determined visually in a glass vessel. This technique, while still commonly used today, is highly subjective, although the author (Zwietering, 1958) claims it to be reproducible to within 2–3% with an experienced observer. The method is highly useful within a narrow range of conditions, but opaque suspensions, high solids volume fractions, and non-transparent tanks, such as stainless steel widely used throughout industry, all render the method unworkable.

Numerous other experimental methods to determine N_{JS} have been developed to remove subjectivity and overcome other limitations (Buurman et al., 1986; Chapman et al., 1983; Einenkel and Mersmann, 1977; Micale et al., 2002; Musil and Vik, 1978; Rao et al., 1988). Each method has its strengths, such as objectivity (Chapman et al., 1983; Musil and Vik, 1978), unobtrusiveness (Einenkel and Mersmann, 1977), or applicability to industrial vessels (Buurman et al., 1986; Musil and Vik, 1978; Rewatkar et al., 1991). Some methods still rely on a visual observation (Einenkel and Mersmann, 1977), which requires a transparent system. Others require the use of an intrusive sensor (Chapman et al., 1983; Musil and Vik, 1978), which might alter the flow regime. Still others are difficult to implement due to unclear criteria, hard to setup correctly, or high experimental variability (Buurman et al., 1986; Musil and Vik, 1978; Rewatkar et al., 1991). Thus far, no single method can combine wide applicability, high accuracy, and ease of use.

CFD methods have been developed that depend on mimicking these sorts of experiments but require unsteady-state simulations that depend on the transient motion of solid particles. Additional CFD methods, which do not directly mimic experimental work, have also been reported. Tamburini et al. (2011) proposed using an unsuspended solids criterion, which is defined by mesh cells exhibiting the maximum packing fraction of solids. Tamburini goes further (Tamburini et al., 2012) to suggest this as a metric for sufficient suspension, which allows for agitation at speeds somewhat lower than complete off bottom suspension.

Mean age theory has gained traction in recent years as a means of acquiring time dependent data from steady state CFD simulations (Gomes et al., 2015; Liu, 2011; Liu and Tilton, 2010; Russ and Berson, 2016). This technique redefines time in a passive, steady state scalar,

* Corresponding author.
E-mail address: eric.berson@louisville.edu (R.E. Berson).

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which then allows for analysis of traditionally time based variables such as mean residence time while using a steady-state CFD solution. Even with the increasing prevalence of high performance and parallel computing capabilities, a steady state solution is more convenient in terms of convergence criteria and speed of computation, especially for long time scale problems and/or larger volume systems. It can, therefore, be highly useful for collecting information such as local residence time (Liu, 2011) or mixing time (Liu, 2011). Mean age theory was originally only applicable to single phase systems, but was recently extended for use in multiphase systems by Russ and Berson (2016).

Multiphase mean age theory (MMA) was applied here to obtain local, time dependent data for solids in proximity to the bottom of mixing vessels, which can be used to define N_{JS}. The objectives were: (1) apply MMA theory to predict N_{JS} in mixing tanks, and (2) validate the technique by comparing computational and experimental N_{JS} across a range of solid densities, solid fractions, and particle size.

2. Theory

Mean age theory is a means of modeling the time dependent behavior of a passive scalar in a steady-state CFD simulation, providing the mean time elapsed for a parcel of material traveling from the system inlet to any other location in the system, accounting for all possible path lines from the inlet to each point. The full derivation has been given extensively elsewhere for single phase (Liu and Tilton, 2010; Sandberg, 1981; Spalding, 1958) and multiphase systems (Russ and Berson, 2016), while a summary of the theory is given here. The theory begins with the advective–diffusive equation:

$$\frac{\partial C}{\partial t} + \nabla \cdot (uC) = \nabla \cdot (D\nabla C) \tag{1}$$

where C is concentration, t is time, u is velocity and D is the diffusion constant. In a multiphase system, C can be defined as:

$$C(x, t) = \rho \times \alpha(x, t) \times \phi(x, t) \tag{2}$$

where ρ is the phase density, α is the phase volume fraction, and ϕ is a dimensionless scalar value for modeling scalar transport. Mean age can be defined as:

$$a(x) = \frac{\int_0^{\infty} tC(x, t)\,dt}{\int_0^{\infty} C(x, t)\,dt} \tag{3}$$

Modifying Eq. (1) and incorporating Eqs. (2) and (3) leads to:

$$\nabla (ua) = \nabla \cdot D\nabla a + 1 \tag{4}$$

where the velocity is that of the phase under examination. Boundary conditions for this system are as follows

$$a = 0 \quad \text{Inlet} \tag{5}$$

$$\frac{\partial a}{\partial x_n} = 0 \quad \text{Outlet} \tag{6}$$

$$\frac{\partial a}{\partial x_n} = 0 \quad \text{Wall} \tag{7}$$

This passive, steady state equation is then solved secondary to the flow solution obtained by CFD to give a spatial distribution of the mean age scalar throughout the entire flow-field.

Because MMA gives time related data, this technique can be extended to analysis of just suspended speed. At its most basic definition, just suspended speed is dependent on the time solids spend in proximity to a vessel bottom (1–2 s (Zwietering, 1958)). Model zone definitions can be applied in conjunction with MMA to allow specific analysis in regions of interest within a vessel. Fig. 1 shows an example of such conditions for examining settling and N_{JS} in a mixing tank.

Zone A was set as a rotating reference frame to adequately accommodate impeller motion for the purpose of modeling. Zones B and C were set as stationary zones in the modeling. Zone C exists solely for examining the age in the bottom of the vessel. The height of Zone C was on the order of 1% of the total height of the vessel, since only the behavior at the very bottom is relevant for determining N_{JS}. The thinner this zone the better, though a lower limit may be reached due to particle size or meshing concerns. The ratio of zone height to particle diameter used here ranged from 15:1 to 60:1.

The scalar age measurement is confined to the solid phase particles. The age of suspended particles (residing anywhere above Zone C) will be relatively low since the transport of the scalar will be convection dominated. However, if the particle is settling (in Zone C), then convective transport of the scalar will go to zero while slow diffusive transport will dominate, which will generate higher order of magnitude age values. Fig. 2 shows a theoretical result for such a condition. Each point represents the average of the MMA in Zone C for a given impeller speed. Above N_{JS}, the age value is very low. Below N_{JS}, the age value is orders of magnitude higher. N_{JS} occurs at the discontinuity between the two regions of the graph. The transition from convection-dominated transport to diffusion-dominated transport as impeller speed decreases results in a very sharp increase in age.

Just suspended speed applications are more typical in closed vessels, so the technique introduced here will be more widely useful if it can also apply to systems with no flow in or out of the vessel. For a closed system, the age transport equation only needs to be solved within Zone C. Both the inlet and outlet boundaries need to be defined at the surface of Zone C where material enters and leaves this zone. Fig. 3 shows how CFD can define where velocity is positive or negative, indicating flow into or flow out of the zone.

Anywhere the z-velocity is negative, the flow is into Zone C and the region can be viewed as an inlet. Anywhere the z-velocity is positive, the flow is out from Zone C and the region

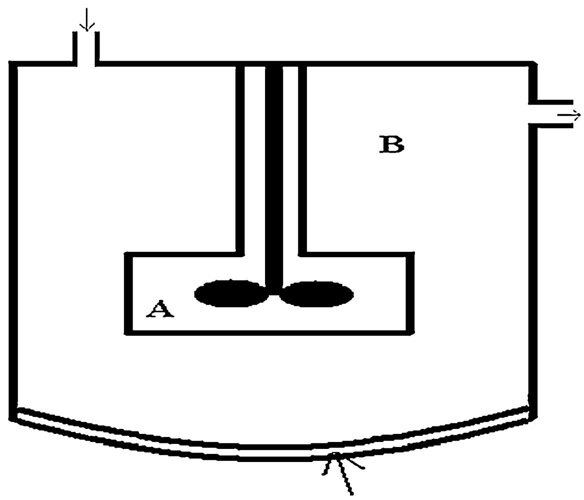

Fig. 1 – Diagram of zone selection for measuring N_{JS}. A is a moving reference frame, B is a stationary zone, and C exists for measuring age near the vessel bottom.