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Inversion of three-dimensional tidal currents in marginal seas by assimilating satellite altimetry

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The four-dimensional variational data assimilation technology based on the theory of inverse problem is applied to simulate the three-dimensional tidal currents in the marginal seas by assimilating the satellite altimetry. The model is calibrated by the twin experiments where the prescribed open boundary conditions for a three-dimensional barotropic tidal model are successfully inverted. By assimilating the tidal harmonic constants derived from TOPEX/Poseidon altimeter data, the open boundary conditions are optimized and the M_2 tidal currents in the Bohai and Yellow Seas (BYS) are simulated in the practical experiment. During the assimilation, the cost function and the gradients of cost function with respect to the open boundary conditions have been decreased significantly. Although the current observations are not assimilated into the model, the cost function composed of the data misfit between model-produced and observed currents is still decreased from 1.00 to 0.09, which demonstrates the reasonability and feasibility of inverting tidal currents from satellite altimetry or other elevation measurements. The co-tidal charts and the near-surface M_2 tidal current ellipses obtained in the practical experiment are in good agreement with the observed tides and tidal currents in BYS.

1. Introduction

The tides and tidal currents are the basic motion forms of ocean water and play an important role in the researching of other processes, such as the storm surge, the circulation and the water mass, etc. [1]. They are especially important in the marginal seas. However, the tides and tidal currents in coastal areas and semi-enclosed seas are often much more difficult to predict and model than those in the deep ocean because of the complex topography [2]. The Bohai Sea and the Yellow Sea (BYS) are typically shallow and semi-enclosed marginal seas. According to the work of Egbert and Ray [3], where the global M_2 tidal energy dissipation was estimated from TOPEX/Poseidon (T/P) altimeter data, the energy dissipation in the Yellow Sea and the East China Sea caused by the tides and tidal currents was very large. Based on their calculation, about 150 GW was dissipated in the Yellow Sea and the East China Sea and only about 50 GW in the South China Sea. Therefore, it is necessary and of great importance to model and predict the tides and tidal currents in BYS accurately. Many works have been devoted to simulating the tides and tidal currents in this area by employing numerical methods (see for example, [4–13]) but most of them were based on two-dimensional (2-D) models and cannot describe the vertical (three-dimensional, 3-D) structure of tidal currents which is becoming more and more important nowadays in order to study the scientific aspects related to marine sediment dynamics [14], transportation of suspended particulate matter [15] and ocean bottom boundary layers [16], etc. In the work of Guo and Yanagi [6], a 3-D tidal current model was applied to the Bohai, Yellow and East China Seas and the major four tides and tidal currents were simulated. Lee and Jung [8] employed a 3-D mode-splitting barotropic model to simulate the M_2 tides and tidal currents in BYECS and four forms of eddy viscosity formulations were compared. Bao et al. [10] simulated the principle four tides and tidal currents in BYECS by using a 3-D turbulent closure model and analyzed the characteristics of tidal currents in detail. Using 3-D mixed finite-difference Galerkin function model, Lee et al. [11] extended the work of Lee and Jung [8] to simulate the oceanic circulation in the Yellow Sea and the East China Sea in the presence of M_2 tide. At present 2-D models are working very well in modeling and predicting the ocean surface elevation. However, because both the quantity and the quality of in situ current observations are not able to satisfy the requirements of researchers, the accuracy of 3-D simulation is still relatively low [17].

Among all the data assimilation methods, four-dimensional variational (4DVAR) data assimilation is one of the most effective and powerful approaches. Based on the theory of inverse problem, it is

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an advanced data assimilation method which involves the adjoint method and has the advantage of directly assimilating various observations distributed in time and space into numerical models while maintaining dynamical and physical consistency with the model. The earlier applications of adjoint assimilation method in oceanography were addressed by Bennett and McIntosh [18] and Prevost and Salmon [19] who applied the weak constraint formalism of Sasaki [20] to a tidal flow problem and a geostrophic flow problem respectively. Thacker and Long [21] then employed the strong constraint formalism in which the model equations were imposed as exact constraints on the minimization. The adjoint method is a powerful tool for parameter estimation [22]. Panchang and O'Brien [23] estimated the bottom drag coefficient in a tidal channel by using some experimental results. The phase speeds were estimated by Smedstad and O'Brien [24] in a reduced gravity model for the tropical Pacific Ocean. The wind stress coefficient and the oceanic eddy viscosity profile were estimated in the work of Yu and O'Brien [25] and the work was extended to optimize the initial condition [26]. By assimilating the velocity data from one or more current meters, Lardner and Das [27] estimated the eddy viscosity profile in a quasi-three-dimensional numerical tidal and storm surge model, and Lardner and Song [28] extended the work to estimate the eddy viscosity profile and friction coefficients. Marotzke et al. [29] constructed the adjoint model of the MITGCM (Massachusetts Institute of Technology Ocean General Circulation Model) by employing R. Giering's software tool Tangent-Linear and Adjoint Model Compiler. More recently, Heemink et al. [30] assimilated tidal gauge data and altimeter data into a 3-D shallow water model. Peng and Xie [31] developed an adjoint model of the 3-D nonlinear POM to construct a 4DVAR algorithm for coastal ocean prediction. Lu and Zhang [13] carried out the twin and practical experiments to invert the spatially varying BFC in a 2-D tidal model by assimilating the Topex/Poseidon (T/P) satellite data and tidal gauge data. In the work of Zhang and Lu [32], by discretizing the primitive equations of motion and continuity on spherical coordinates and its adjoint equations, a 3-D numerical barotropic adjoint tidal model was developed and three kinds of model parameters, i.e. the open boundary conditions, the bottom friction coefficients and the vertical eddy coefficients, were estimated with the help of twin experiments. The previous works mainly discussed the parameter estimation problems for 3-D tidal models by using the twin experiments (see for example [27,28,32]) and were deficient in practical application. The work of this paper is the extension of Zhang and Lu [32] and the M_2 tide and tidal current in BYS will be simulated by assimilating the satellite altimetry.

The sea surface height (SSH) data from which the tidal harmonic constants are derived are mostly measured by tidal gauge stations located near the coast or the islands, so the observations are usually sparse in space. A large amount of SSH data with high accuracy has been provided since the launch of T/P satellite on August 10, 1992 by NASA and CNES. Satellite altimetry has reopened the problem of how tidal dissipation is to be allocated [1] and led the ocean research to 'an age of altimetry' [17]. By assimilating harmonic constants from T/P altimetry and tidal gauge stations, the adjoint method has been widely used in the 2-D simulation of tides in BYECS (see [12,13]). However, as presented in the last paragraph, because the structure of tidal currents and the ocean topography in marginal seas are usually quite complex, the application of adjoint method in 3-D simulation of tidal currents are mainly concentrated on parameter estimation problems by employing the twin experiments. Obviously, the in situ observing is the most direct and effective method to study the vertical structure of tidal currents. However, it is strictly restricted by the cost and observing techniques. At present the observing techniques of SSH have been quite efficient and researchers have possessed a great number of SSH measurements with good quality and continuity, especially after the emergence of satellite altimetry [17]. How to invert the current fields from these SSH measurements is an interesting and meaningful problem which has been studied and debated for decades. One method is to directly solve the depth-integrated momentum equations through using local elevation gradients [33,34] which is easy to realize, but the calculation results will be very sensitive to the error of SSH observations. As well, the resulting current fields may fail to satisfy mass conservation law and the method cannot be performed at the tidal critical latitude [17]. Another approach is to employ data assimilation techniques in numerical ocean models. McIntosh and Bennett [35] and Egbert et al. [36] carried out the application of Generalized Inverse Methods in tidal models. Kantha [37] applied nudging techniques into a fully nonlinear barotropic tidal model of the global oceans to derive tides from T/P altimetry. A clear advantage of data assimilation techniques is their explicit accounting for errors in both tidal observations and assumed dynamics [17]. Data assimilation has shown little advantage in global scale tidal calculation [38] but the advantage is obvious for regional scales [17]. In the work of Ray [17], an intermediate method was put forward and barotropic tidal currents were inverted from measured elevations by solving the 2-D momentum equations and the continuity equation in the least squares fashion. This method which is called the least squares inversion approach has been employed to map empirically the open-ocean tidal dissipation [39]. It should be noted that the works listed above were all based on 2-D (depth-integrated) equations. In this paper, the adjoint method will be applied into a 3-D barotropic tidal model to invert the 3-D tidal currents by assimilating T/P satellite altimetry. After assimilation, both the cost function and the gradients of cost function with respect to the OBCs were decreased significantly. The co-tidal charts and the near-surface M_2 tidal current ellipses obtained in the practical experiment were in good agreement with the observed tide and tidal current in BYS.

This paper is organized as follows. A brief introduction is given to the 3-D adjoint tidal model in Section 2. The identical twin experiments are carried out in Section 3 and the results are analyzed. In Section 4, the M_2 tide and tidal current in BYS are simulated by assimilating the T/P satellite altimetry. The co-tidal charts and the near-surface M_2 tidal current ellipses are presented. Finally, Section 5 provides summary and main conclusions and completes the paper.

2. Tidal model

2.1. Governing equations

Assuming that the pressure is hydrostatic and the density is constant, the 3-D, nonlinear, time-dependent, free surface, primitive equations of motion and continuity on spherical coordinates are given as follows.

$$\frac{\partial u}{\partial t}+\frac{u}{a}\frac{\partial u}{\partial \lambda}+\frac{v}{R}\frac{\partial u}{\partial \phi}+w\frac{\partial u}{\partial z}-\frac{uv\tan\phi}{R}-fv+\frac{g}{a}\frac{\partial \zeta}{\partial \lambda} -A_h\Delta u-\frac{\partial}{\partial z}\left(A_v\frac{\partial u}{\partial z}\right), \tag{1.1}$$

$$\frac{\partial v}{\partial t}+\frac{u}{a}\frac{\partial v}{\partial \lambda}+\frac{v}{R}\frac{\partial v}{\partial \phi}+w\frac{\partial v}{\partial z}+\frac{u^2\tan\phi}{R}+fu+\frac{g}{R}\frac{\partial \zeta}{\partial \phi} -A_h\Delta v-\frac{\partial}{\partial z}\left(A_v\frac{\partial v}{\partial z}\right)=0, \tag{1.2}$$

$$\frac{\partial w}{\partial z}+\frac{1}{a}\frac{\partial u}{\partial \lambda}+\frac{1}{a}\frac{\partial (v\cos\phi)}{\partial \phi}=0, \tag{1.3}$$

where t is the time, λ and ϕ are the east longitude and north latitude, respectively, ζ is the sea surface elevation above the undisturbed sea level, $u(x,y,z,t)$, $v(x,y,z,t)$ and $w(x,y,z,t)$ are the velocity