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On the adjoint-consistent formulation of interface conditions in goal-oriented error estimation and adaptivity for fluid–structure interaction

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ABSTRACT

The numerical solution of fluid–structure-interaction problems poses a paradox in that most of the computational resources are consumed by the subsystem of least practical interest, viz., the fluid. Goal-oriented adaptive discretization methods provide a paradigm to bypass this paradox. Based on the solution of a dual problem, the contribution of local residuals to the error in a specific goal functional is estimated, and only the regions that yield a dominant contribution are refined. In the present work, we address a fundamental complication in the application of goal-oriented adaptivity to fluid–structure-interaction problems, namely, that the treatment of the interface conditions has nontrivial consequences for the properties of the dual problem. In the context of a linearized model problem, we consider two equivalent discretizations differing only on the formulation of the interface coupling terms. By means of an adjoint consistency analysis, we show that only one of these discretizations is adjoint consistent. Numerical experiments convey that the two discretizations behave very differently for the dual problem, and that the adjoint-consistent discretization yields more reliable error estimates. Based on the adjoint-consistent discretization, we finally present some *h*- and *hp*-adaptive results, confirming that tremendous savings in computational cost can be realized through the use of goal-oriented refinement strategies. The numerical experiments illustrate that the goal-oriented approach effectively equilibrates the error contributions of the fluid and structure subsystems, which is imperative for efficiently resolving the coupled fluid–structure-interaction problem, and which cannot be accomplished by uniform or residual-based refinement strategies.

1. Introduction

The numerical simulation of fluid–structure-interaction problems generally requires vast computational resources. An interesting paradox is that the computational work is dominated by the complexity of the subsystem that is of least practical interest, viz., the fluid. Indeed, the fluid consumes nearly all of the computational resources while quantitative concern is primarily restricted to the structural response. For instance, in [11] it is reported that for the identification of the aeroelastic mode shapes of an F-16 aircraft more than 95% of the total simulation time is spent inside the fluid solver and the fluid-mesh update algorithm.

Substantial savings in computational cost can be realized by optimizing the computational effort according to the specific goals of the simulation. We note that in many engineering applications the aim is not to determine the solution itself, but rather to evaluate specific functionals of the solution, so-called *goal*, *target* or *output functionals*. Meaningful goal functionals in the context of fluid–structure interactions include, for instance, the global forces exerted by the fluid on the structure [37], the stresses and displacements induced at specific points in the structure [46,39], or the net energy that is transferred from the fluid to the structure [16]. The crucial point is that in order to reliably determine one such goal functional, it is generally not necessary to fully resolve all the small-scale features in the fluid and structure subsystems. Substantial savings in computational cost can thus be realized by resolving only those features that bear a pronounced influence on the goal quantity of interest.

In general, it may be difficult to anticipate the local levels of resolution needed in the fluid and structure approximation spaces to accomplish this. Heuristic approaches, such as a priori mesh refinement in the vicinity of the fluid–structure interface, are likely to fail resulting in a costly and possibly even inaccurate approximation of the goal functional. The use of adaptive discretization techniques, capable of providing automated control of the goal-quantity error to within a user-defined tolerance, is therefore

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indispensible for the efficient and reliable computation of complex fluid–structure-interaction problems.

Such adaptive discretization techniques rely on *goal-oriented error estimates* for the output functional of interest. The derivation of these error estimates proceeds a posteriori by a duality argument and requires the solution to an appropriate dual (linearized-adjoint) problem. Refinement indicators are obtained by identifying element-wise contributions to the error estimate. Accordingly, a *goal-oriented adaptive refinement strategy* can be devised, in which, starting from a very coarse discretization, only those regions in the fluid and structure mesh are refined that contribute significantly to the error in the goal functional under consideration.

Pioneering work in the field of goal-oriented error estimation and adaptivity has been performed by Becker and Rannacher [3] and Prudhomme and Oden [35]; see in particular the comprehensive overviews [4,15,36] and the references therein. For related work in the context of discontinuous Galerkin methods we also refer to the efforts by Houston and Süli [24,38] and Hartmann and Houston [20,21]. As it stands, the existing framework provides a systematic approach for deriving a posteriori error estimates in general output functionals of interest, which applies generically to both linear and non-linear (initial-)boundary-value problems. Its performance has already been demonstrated for a wide variety of applications. The extension to multiscale and multiphysics problems, however, is still in a relatively early stage; see the latest works of Larson et al. [27–30], Estep et al. [5,9,10] and Prudhomme and Bauman et al. [2,34]. First applications of goal-oriented adaptivity to fluid–structure-interaction problems are just starting to emerge and can be found in the works of Dunne [7], Grätsch and Bathe [17], and Van der Zee et al. [41–43]; see also [13] for related work on norm-oriented adaptivity using explicit residual-based error estimators.

When applying the standard framework for goal-oriented error estimation and adaptivity to fluid–structure-interaction problems, several complications arise. First, the manner in which the free-boundary character manifests itself in the dual problem is nontrivial and gives rise to the occurrence of shape derivatives. Some authors have therefore resorted to approaches that bypass a proper derivation of the dual problem, either by considering a fully Eulerian formulation as in [7] or by using a crude finite-difference approximation of the linearized-adjoint [17]. In contrast to this, in the recent work of Van der Zee et al. [41,42,44,45], the linearization of the domain-dependent non-linearity is no longer bypassed, but rigorously pursued using ideas from shape differential calculus.

Second, the treatment of interface conditions in the primal formulation of the fluid–structure-interaction problem, including the enforcement of flow tangency conditions for the fluid subproblem and the evaluation of load functionals for the structure subproblem, generally has nontrivial consequences for the well-posedness of the dual problem. In particular, due to the swapping of trial and test spaces between the primal and dual problem, the role of the coupling terms is interchanged at the fluid–structure interface. Thereby, coupling terms that are present in the variational formulation of the primal fluid subproblem reappear in the variational formulation of the dual structure subproblem, and vice versa. This may give rise to an *adjoint-inconsistent* treatment of the interface conditions, rendering the dual variational formulation an inconsistent representation of the underlying adjoint initial-boundary-value problem. The quality of the discrete approximation of the dual solution can be greatly affected by this, as well as the a posteriori error estimates that are derived from it.

In this paper, we address the second of the aforementioned issues, i.e., the treatment of interface conditions and its effect on the dual problem. For this purpose, any issues related to the free-boundary character of the fluid–structure interface are deliberately bypassed by introducing a geometric linearization. By means of an adjoint consistency analysis along the lines of [19], it is shown that two distinct formulations that appear to be equivalent for the primal problem, can behave very differently for the dual problem. In particular, we highlight that a straightforward and seemingly correct treatment of the interface coupling conditions in the primal problem, may give rise to inconsistent solutions for the dual problem that exhibit irregularities at the fluid–structure interface. We present numerical examples that demonstrate the effect of this on the accuracy of the computed goal functionals and the sharpness of the goal-oriented error estimates and bounds. It is concluded that the adjoint-consistent treatment of interface conditions is crucial for the accurate and reliable a posteriori estimation of errors in output quantities of interest; cf. [18].

The remainder of this paper is organized as follows. We start in Section 2 by introducing a geometrically linearized model problem and stating some relevant goal functionals. Next, in Section 3 we discretize the model problem using a space–time finite element method for both the fluid and structure. Two discretizations will be presented arising from different formulations of the interface coupling terms. Subsequently, in Section 4 we derive dual-weighted a posteriori error estimates for generic (non-)linear output functionals of interest. Adjoint consistency of the discretization is discussed and analyzed in Section 5. Then, in Section 6 we present some numerical examples, highlighting the importance of adjoint consistency and demonstrating the use of goal-oriented error estimates to guide an *hp*-adaptive mesh refinement strategy. Finally, we end in Section 7 with some concluding remarks.

2. Problem statement

As a model problem, we consider a geometrically linearized version of the two-dimensional panel problem from Piperno and Farhat [33], pertaining to the aeroelastic response of a flexible beam immersed in an inviscid compressible fluid flow; see Fig. 1. Below, we briefly introduce the governing initial-boundary-value problems for the fluid and the structure, as well as the interface conditions interconnecting the two. In addition, we present several relevant goal functionals.

2.1. Fluid subproblem

For the formulation of the fluid subproblem, we consider the space–time domain $Q_f := \Omega \times I$, consisting of the open bounded domain $\Omega \subset \mathbb{R}^2$ extruded in time over the interval $I = (0,T)$. The boundary of Q_f is denoted by ∂Q_f and consists of the lower time boundary $\Omega \times \{0\}$, the upper time boundary $\Omega \times \{T\}$ and the lateral

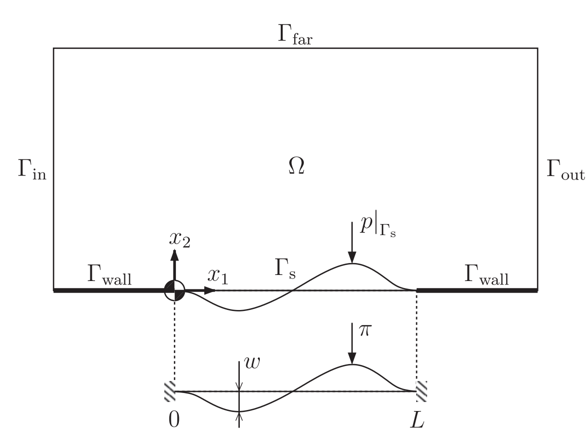

Fig. 1. Illustration of the model problem with an expanded view of the interface region.