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Accurate estimation of evolutionary power spectra for strongly narrow-band random fields

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A B S T R A C T

One of the most widely used techniques for the simulation of non-homogeneous random fields is the spectral representation method. Its key quantity is the power spectrum, which characterizes the random field in terms of frequency content and spatial evolution in a mean square sense. The paper at hand proposes a method for the estimation of separable power spectra from a series of samples, which combines accurate spectrum resolution in space with an optimum localization in frequency. For non-separable power spectra, it can be complemented by a joint strategy, which is based on the partitioning of the space-frequency domain into several sub-spectra that have to be separable only within themselves. Characteristics and accuracy of the proposed method are demonstrated for analytical benchmark spectra, whose estimates are compared to corresponding results of established techniques based on the short-time Fourier, the harmonic wavelet and the Wigner–Ville transforms. It is then shown by a practical example from stochastic imperfection modeling in structures that in the presence of strong narrow-bandedness in frequency, the proposed method for separable random fields leads to a considerable improvement of estimation results in comparison to the established techniques.

1. Introduction

Within the last three decades, computational stochastic mechanics has evolved into a self-contained and prolific field of research, which has brought forth a wide range of sophisticated and well-established methodologies for the stochastic simulation of uncertain engineering systems [6,8,33,40,42]. Due to the rapidly growing availability of large-scale and cheap computer power, the intensive computational demands of these techniques become more and more manageable today, which makes stochastic simulation increasingly interesting for actual use in engineering practice. One emerging field of application is the stability analysis of thin-walled structures [7,40], where the random variability of geometric and material imperfections leads to considerable uncertainty in corresponding buckling loads. As a starting point, a series of computational studies has recently shown that the influence of random imperfections on the size and variability of the ultimate strength of cylindrical shells can be reproduced with respect to corresponding experimental tests [1,23–26,32–34,38].

Apart from algorithmic maturity, the quality of stochastic simulation methods predominantly depends on the accurate reproduction of the random physical key phenomena by corresponding random field models. In the case of imperfection triggered buckling, a small misrepresentation of the physical imperfection wave length in the imperfection model can lead to a large discrepancy between real and simulated system response, because the dominant buckling mode might shift to a higher Eigenform, leading to an unphysical increase in ultimate strength. One of the most widely used techniques for the simulation of imperfections as random fields is the spectral representation method [9,35,36,39]. The key quantity of spectral representation is the power spectrum [18,27,28,30,31,44], which is related to the average energy of the random field and is obtained for example in earthquake applications by estimation from measured ground motion accelerations [4,12,37]. Despite its decisive importance for realistic stochastic buckling simulations, only little experience exists so far in transferring experimental imperfection measurements, which typically are strongly narrow-band functions at very low frequencies, into accurate evolutionary power spectra. Up to now, measurement based evolutionary imperfection modeling relies on the adoption of established time–frequency analysis techniques from digital time signal processing [24,25].

Against this background, the present paper intends to shed some light on key issues related to the evolutionary power spectrum estimation of strongly narrow-band random fields, with special emphasis on their application to imperfection modeling in structures. First, a concise review of existing methods for the estimation of evolutionary power spectra [2,3,27] is presented. Second, a simple yet effective method for evolutionary spectrum

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estimation of separable random fields is introduced, referred to as method of separation in the following, which provides accurate spectrum resolution in space at an optimum localization in frequency. Third, a joint strategy for non-separable fields is proposed, which is based on partitioning the space-frequency domain into several sub-spectra that have to be separable only within themselves and can thus be treated consecutively by the method of separation. Fourth, the presented methods are applied for the estimation of benchmark spectra with different bandwidths and frequency content, comprising a modulated Kanai–Tajimi spectrum [13,37] and a practical example of geometric imperfections in an I-section flange [11]. The results demonstrate weaknesses of the established techniques, i.e. the limitation of simultaneous space-frequency localization or the appearance of negative spectral density, and the advantages of the proposed method in terms of accurate space-frequency localization of its spectrum estimates, which is of particular importance in the presence of strong narrow-bandedness in frequency.

The present paper is organized as follows: Section 2 briefly summarizes relevant elements of stochastic process theory. Sections 3 and 4 contain a short overview of established space-frequency analysis techniques and an in-depth derivation of the method of separation, respectively. Section 5 illustrates difficulties of evolutionary spectrum estimation in the presence of strong narrow-bandedness and shows the results of each method for the imperfection example.

2. Some elements of stochastic process theory

A random field $h(x)$, equivalently known as a stochastic process in a time–frequency context, represents an ensemble of spatial functions, whose exact values are a priori indeterminate, but follow a predefined probability distribution [18,27,44]. It can be split into a deterministic mean $\mu(x) = E[h(x)]$ and a zero-mean random field $f(x) = h(x) - \mu(x)$. The stochastic structure of $f(x)$ is characterized by a number of higher moments, starting with the variance or mean square $E[|f(x)|^2]$, and an autocorrelation function $R(x,\tau)$, which determines for any $f(x)$ the dependence of neighbouring values as a function of their spatial distance τ [18,27,44]. The operator $E[]$ denotes mathematical expectation, which can be evaluated by simple ensemble averaging [18].

The Fourier transform $F(\omega)$, which decomposes a zero-mean random field $f(x)$ of length L by projecting it onto the basis of sines and cosines as a function of frequency ω [14,18,22], reads

$$F(\omega) = \frac{1}{2\pi} \cdot \int_0^L f(x) \cdot e^{-I\omega x}\, dx, \tag{2.1}$$

where I denotes the imaginary unit. The transformation of Eq. (2.1) can be evaluated in discrete form by the computationally efficient Fast Fourier Transform (FFT) [14,18]. In view of its trigonometric decomposition, $f(x)$ can be completely characterized by a two-sided power spectrum S [18,27,28,30,31,44], which is called homogeneous, if $S(\omega)$ depends only on frequency ω, and evolutionary, if $S(\omega,x)$ depends on both frequency ω and space x. Mathematically, the power spectrum is defined as the Fourier transform of the autocorrelation function $R(x,\tau)$ (Wiener–Khintchine theorem) [30,31]. An intuitive approach to the power spectrum is provided by its interpretation as the distribution of the mean square of the random field $f(x)$ over the space-frequency domain, so that it holds

$$E[|f(x)|^2] = 2\int_0^\infty S(\omega,x)\, d\omega. \tag{2.2}$$

In this context, Eq. (2.2) is also denoted as the incremental energy or instantaneous power in space. Analogous to Eq. (2.2), the incremental energy in frequency is defined as

$$E[|F(\omega)|^2] = \int_0^L S(\omega,x)\, dx. \tag{2.3}$$

Eqs. (2.2) and (2.3) are also known as the marginal spectral densities of a random field [3]. The power spectrum is called narrow-band, if the bulk of its energy is located only within a very small frequency band [18]. Additionally, the power spectrum satisfies spectral separability, if it can be multiplicatively decomposed into a homogeneous spectrum part $S(\omega)$ and a modulating spatial envelope $g(x)$ as

$$S(\omega,x) = S(\omega) \cdot g(x). \tag{2.4}$$

The corresponding random field is then called separable.

If the power spectrum $S(\omega,x)$ of a random field is known, an arbitrary number m of corresponding Gaussian random samples can be generated by the spectral representation method [9,35,36,39], which reads for a one-dimensional univariate zero-mean Gaussian random field

$$f^{(i)}(x) = \sqrt{2}\sum_{n=0}^{N-1} A_n \cos\left(\omega_n x + \phi_n^{(i)}\right), \tag{2.5}$$

with

$$A_n = \sqrt{2 \cdot S(\omega_n, x) \cdot \Delta\omega}, \tag{2.6a}$$
$$\omega_n = n \cdot \Delta\omega, \tag{2.6b}$$
$$\Delta\omega = \omega_{up}/N, \tag{2.6c}$$
$$A_0 = 0 \text{ or } S(\omega_0 = 0, x) = 0, \tag{2.6d}$$

where $i = 1,2,\ldots,m$ and $n = 0,1,2,\ldots,(N-1)$. The parameter ω_{up} is the cut-off frequency, beyond which the power spectrum is assumed to be zero, the integer N determines the discretization of the active frequency range, and $\phi_n^{(i)}$ denotes the $(i)^{th}$ realization of N independent phase angles uniformly distributed in the range $[0, 2\pi]$. For non-Gaussian random fields, the translation field theory can be used to generate random samples from a simple transformation of an underlying Gaussian field [10,40].

The performance of the evolutionary spectrum estimation methods to be presented in the following is tested by a uniformly modulated Kanai–Tajimi spectrum, which is defined according to Eq. (2.4) by its separable components

$$S(\omega) = \frac{1 + 4\varsigma^2\left(\frac{\omega}{\omega_0}\right)^2}{\left[\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\varsigma\frac{\omega}{\omega_0}\right)^2\right]}, \tag{2.7}$$

$$g(x) = \frac{e^{-0.25x} - e^{-0.5x}}{0.25}. \tag{2.8}$$

Parameters $\omega_0 = 10$ rad/mm and $\varsigma = 0.24$ represent the natural frequency and the damping ratio, respectively. The Kanai–Tajimi spectrum of Eq. (2.7) has been widely applied in a time–frequency context for the stochastic simulation of seismic ground accelerations, and various modulating terms leading to both separable and non-separable spectra can be found in the literature [13,16]. The specific values for ω_0 and ς in conjunction with the exponential modulating function of Eq. (2.8) are adopted from [37] and yield a power spectrum with equally pronounced evolution in space and frequency directions, therefore representing a suitable benchmark for evolutionary estimation techniques (see Figs. 1 and 2). In view of the energy interpretation of the spectrum, Eq. (2.7) can be conceived of as the incremental energy distribution in frequency direction, which does not change its shape, but is merely modulated in amplitude along the spatial axis by Eq. (2.8).

For a performance test in the non-separable case, a composed benchmark spectrum is defined as