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A hybrid finite element-scaled boundary finite element method for crack propagation modelling

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ABSTRACT

This study develops a novel hybrid method that combines the finite element method (FEM) and the scaled boundary finite element method (SBFEM) for crack propagation modelling in brittle and quasi-brittle materials. A very simple yet flexible local remeshing procedure, solely based on the FE mesh, is used to accommodate crack propagation. The crack-tip FE mesh is then replaced by a SBFEM rosette. This enables direct extraction of accurate stress intensity factors (SIFs) from the semi-analytical displacement or stress solutions of the SBFEM, which are then used to evaluate the crack propagation criterion. The fracture process zones are modelled using nonlinear cohesive interface elements that are automatically inserted into the FE mesh as the cracks propagate. Both the FEM's flexibility in remeshing multiple cracks and the SBFEM's high accuracy in calculating SIFs are exploited. The efficiency of the hybrid method in calculating SIFs is first demonstrated in two problems with stationary cracks. Nonlinear cohesive crack propagation in three notched concrete beams is then modelled. The results compare well with experimental and numerical results available in the literature.

1. Introduction

Since the smeared [1] and discrete [2] crack models were introduced in the late 1960s, they have been extensively used to model crack propagation in brittle and quasi-brittle materials. In smeared crack models the cracks which are "smeared" out over the domain are represented by softening stress–strain relationships in the normal and tangential directions of the crack surfaces. Discrete crack models, on the other hand, represent cracks by separating the nodes along the crack path and usually needs remeshing. Although the smeared crack models have been more popular due to their ease in implementation, they suffer from spurious stresses due to the kinematic incompatibility between elements and have to be remedied by introducing a shear retention factor [3] or by rotating the cracks so that they are aligned with the principal stress and strain directions [4]. Recent approaches to model discontinuities caused by cracks enrich either the elements cut by the crack (embedded crack models e.g., [5–7]) or the nodes along the crack path (extended finite element method (XFEM) e.g., [8–10]) using discontinuous functions. These approaches do not need remeshing and have shown tremendous potential in modelling crack propagation.

Discrete crack models use remeshing algorithms to update mesh topology as cracks propagate. They require more implementation effort. However, they are capable of accurately representing the discontinuities between the crack surfaces, and are useful to study the local behaviour in the vicinity of cracks. Most of the discrete crack models are based on the finite element method (FEM). As stress singularities at crack tips are not accounted for by conventional FEM formulations, very fine crack tip meshes are often needed to accurately calculate fracture parameters such as the stress intensity factors (SIFs), energy release rates and crack-tip stresses, which are used in crack propagation criteria. Various remedies have been proposed, such as the quarter-point elements [11], hybrid Trefftz elements [12], elements having shape functions with crack tip asymptotic expansions [13], enriched elements [14] and hybrid crack elements (HCE) [15–18]. In general, however, these remedial methods still need fine crack-tip meshes or impose special requirements on the shape and element types of crack-tip meshes. As a result, the remeshing algorithms used in FEM to accommodate crack propagation are usually very complicated.

Recently, Yang et al. [19–21] successfully applied the scaled boundary finite element method (SBFEM) to model discrete crack propagation in concrete structures. The SBFEM is a semi-analytical method developed by Wolf and Song [22] and is very efficient in solving problems with unbounded media, discontinuities and singularities. In modelling crack propagation problems, the SBFEM can extract accurate SIFs from semi-analytical solutions of displacements or stresses using substantially fewer degrees-of-freedom (DOF) than the FEM. Moreover, the remeshing and mesh mapping procedures in the SBFEM are much simpler yet

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more accurate. The SBFEM-based methods developed in [19–21] are particularly suitable for problems with single crack or a few cracks. For problems with many cracks that become too close during propagation, the remeshing operation is cumbersome because the subdomains may become so distorted that not all the nodes are visible from their scaling centres.

To tackle this problem, this study further develops a hybrid method that combines the FEM and the SBFEM. It can model crack propagation in both linear elastic- and nonlinear-fracture mechanics problems. In the latter, cohesive interface elements (CIEs) are automatically inserted into the FE mesh as the cracks propagate. A very simple yet flexible local remeshing procedure, based on the FE mesh only, is used to accommodate crack propagation. The crack-tip FE mesh is then replaced by a SBFEM rosette to extract accurate SIFs, which are subsequently used to evaluate the crack propagation criterion. Both the FEM's flexibility in remeshing for multiple crack problems and the SBFEM's high accuracy in calculating SIFs are thus exploited.

2. The hybrid finite element-scaled boundary finite element method

The basic idea of this method is to carry out remeshing solely based on the FE mesh, and to calculate the SIFs from the SBFEM rosette which replaces the crack-tip FE mesh. The key elements of the method are described below.

2.1. The scaled boundary finite element method

In the SBFEM, a domain is partitioned into a number of subdomains. Fig. 1a shows a typical two-dimensional SBFEM mesh with three subdomains, in which subdomain 1 contains a crack (cracked subdomain). Only the boundaries of subdomains are discretized using one-dimensional finite elements. The crack surfaces are not discretized. Each subdomain has a scaling centre (x_0, y_0) from which all the nodes on the boundary of the subdomain are visible. In the cracked subdomain, the scaling centre is positioned at the crack tip. The coordinates of any point in a subdomain are uniquely defined by a local coordinate system (ξ, s). ξ is the radial coordinate which varies from zero at the scaling centre to one on the subdomain boundary. s is the circumferential coordinate (see Fig. 1b). The local coordinates (ξ, s) are related to the Cartesian coordinates (x, y) by the scaling equations [23]

$$x = x_0 + \xi\left(\frac{x_1 + x_2 - 2x_0}{2} + \frac{(x_2 - x_1)s}{2}\right) = x_0 + \xi x_s(s), \tag{1}$$

$$y = y_0 + \xi\left(\frac{y_1 + y_2 - 2y_0}{2} + \frac{(y_2 - y_1)s}{2}\right) = y_0 + \xi y_s(s), \tag{2}$$

(x_1, y_1) and (x_2, y_2) are the nodal coordinates of an element on the boundary.

In each subdomain, an approximate solution is sought in the form

$$\mathbf{u}(\xi, s) = \mathbf{N}(s)\mathbf{u}(\xi), \tag{3}$$

where $\mathbf{u}(\xi)$ is the displacement function in the radial direction and $\mathbf{N}(s)$ is the shape function matrix. For a two-noded element on the subdomain boundary

$$\mathbf{N}(s) = \begin{bmatrix} N_1 & 0 & N_2 & 0 \\ 0 & N_1 & 0 & N_2 \end{bmatrix} \tag{4}$$

with

$$N_1 = \frac{1}{2}(1 - s), \quad N_2 = \frac{1}{2}(1 + s). \tag{5}$$

Substituting Eq. (3) into the principle of virtual work, the governing equations of SBFEM for a subdomain can be obtained as [23]

$$\mathbf{p} = \mathbf{E^0}\mathbf{u}(\xi)_{,\xi} + \mathbf{E^1}^T\mathbf{u}(\xi)\Big|_{\xi=1} \tag{6}$$

$$\mathbf{E^0}\xi^2\mathbf{u}(\xi)_{,\xi\xi} + (\mathbf{E^0} + \mathbf{E^1}^T - \mathbf{E^1})\xi\mathbf{u}(\xi)_{,\xi} - \mathbf{E^2}\mathbf{u}(\xi) = 0, \tag{7}$$

where

$$\mathbf{p} = \int_s \mathbf{N}(s)^T\mathbf{t}\, ds, \tag{8}$$

$$\mathbf{E}^0 = \int_s \mathbf{B}^1(s)^T\mathbf{D}\mathbf{B}^1(s)|J|ds, \tag{9}$$

$$\mathbf{E}^1 = \int_s \mathbf{B}^2(s)^T\mathbf{D}\mathbf{B}^1(s)|J|ds, \tag{10}$$

$$\mathbf{E}^2 = \int_s \mathbf{B}^2(s)^T\mathbf{D}\mathbf{B}^2(s)|J|ds. \tag{11}$$

Eqs. (6) and (7) ensure equilibrium is satisfied analytically in the radial direction and in the finite element sense in the circumferential direction. In Eqs. (8)–(11), $\mathbf{p}$ is the nodal load vector due to boundary tractions, $\mathbf{E}^0, \mathbf{E}^1$ and $\mathbf{E}^2$ are coefficient matrices that depend on the geometry and material properties of the subdomain,

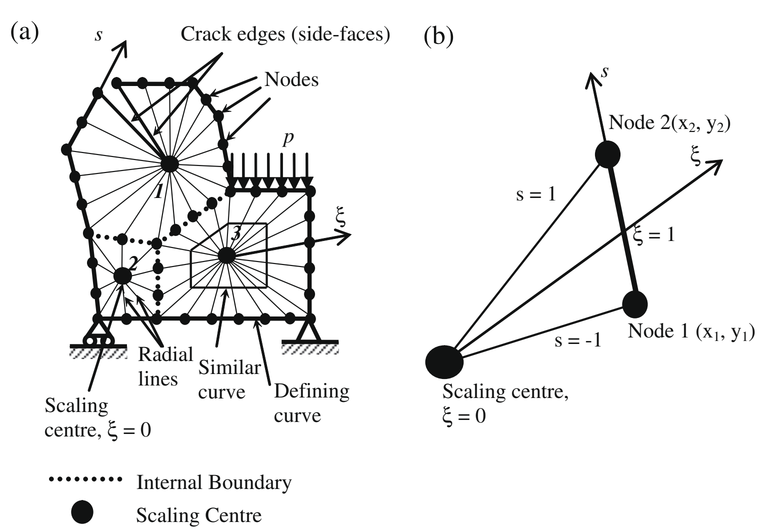

Fig. 1. (a) A typical SBFEM mesh and (b) a two-noded line element on subdomain boundary.