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Nonlinear buckling optimization of composite structures

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ABSTRACT

The paper presents an approach to nonlinear buckling fiber angle optimization of laminated composite shell structures. The approach accounts for the geometrically nonlinear behaviour of the structure by utilizing response analysis up until the critical point. Sensitivity information is obtained efficiently by an estimated critical load factor at a precritical state. In the optimization formulation, which is formulated as a mathematical programming problem and solved using gradient-based techniques, a number of the lowest buckling factors are included such that the risk of "mode switching" during optimization is avoided. The presented optimization formulation is compared to the traditional linear buckling formulation and two numerical examples, including a large laminated composite wind turbine main spar, to clearly illustrate the pitfalls of the traditional formulation and the advantage and potential of the presented approach.

1. Introduction

The use of fiber-reinforced polymers has gained an ever-increasing popularity due to their superior mechanical properties. Designing structures made out of composite material represents a challenging task, since both thicknesses, number of plies in the laminate and their relative orientation must be selected. The best use of the capabilities of the material can only be gained through a careful selection of the layup. This work focuses on optimal design of laminated composite shell structures i.e. the optimal fiber orientations within the laminate which is a complicated problem. One of the most significant advances of optimal design of laminate composites is the ability of tailoring the material to meet particular structural requirements with little waste of material capability. Perfect tailoring of a composite material yields only the stiffness and strength required in each direction. A survey of optimal design of laminated plates and shells can be found in [1].

Stability is one of the most important objectives/constraints in structural optimization and this also holds for many laminated composite structures, e.g. a wind turbine blade. Traditionally in optimization, stability is regarded as the linear buckling load, but for structures exhibiting a nonlinear response when loaded the traditional approach can lead to unreliable design results, see e.g. [2]. In stability analysis the buckling load is often approximated by linearized eigenvalue analysis at an initial prebuckling point (linear buckling analysis) and the buckling load is generally overestimated. In the case where nonlinear effects cannot be ignored nonlinear path tracing analysis is necessary. For limit point instability, several standard finite element procedures allow the nonlinear equilibrium path to be traced until a point just before the limit point. The traditional Newton like methods will probably fail in the vicinity of the limit point and the post-critical path cannot be traced. More sophisticated techniques, as the arc-length methods suggested by [3] and subsequently modified by [4] and [5]are among some of the techniques available today for path tracing analysis in the post-buckling regime.

A more accurate estimate of the buckling load, than that obtainable with linear buckling, can be obtained by performing a geometrically nonlinear response analysis and approximate the buckling load by an eigenvalue analysis on the deformed configuration. Various eigenvalue problems have been suggested for the stability analysis of nonlinear structures. [6] and [7] formulated linear eigenvalue problems with information at one load step on the nonlinear prebuckling path. This formulation is referred as the "one-point" approach, where stiffness information is extrapolated until a singular tangent stiffness is obtained. [8] formulated a linear eigenvalue problem utilizing tangent information at two successive load steps on the nonlinear prebuckling path, and are referred as the "two-point" approach.

Optimization with stability constraints has been studied extensively in the past. [9] and [10] described an optimality criterion method for determining the minimum weight design of linear space truss structures subjected to stability constraints. They solved linear stability analysis problems to obtain the critical load and obtained sensitivities by differentiating the discretized matrix eigenvalue problem with respect to design variables. Later methods for obtaining optimum designs of truss structures with stability constraints while considering geometric nonlinearities were presented by [11] by using a relation based on equal strain energy density in all members.

[12] presented design sensitivities of the buckling load for nonlinear structures by taking derivatives of discretized matrix equations with respect to design variables. The method only works for limit points and the critical point needs to be precisely determined for evaluation of

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sensitivities. [13] presented a variation of the formula that would not only work for limit points but also for bifurcation points.

[14] presented a formulation of continuum design sensitivity analysis of the critical load based on the "one-point" and "two-point" linearized eigenvalue problem. Their expressions would work at any prebuckling point on the nonlinear equilibrium path. They noted that the design sensitivities did not converge to those of the exact critical load when approximated in the near vicinity of the critical point due to divergence in the derivatives of the displacements.

[15] approximated the exact design sensitivities derived by [12] by applying the concept from nonlinear stability analysis, either by "one-point" or "two-point" approach. It was noted that the approximated design sensitivities converged to those by [12] when the approximation point approaches the exact critical point. [16] adopted the method by [15] and included imperfections for avoidance of bifurcation points.

Research on the subject of structural optimization of composite structures considering stability has been reported by many investigators. The first work to appear concerned simple composite laminated plates and circular cylindrical shells where stability was determined by solution of buckling differential equations, see [17–26]. Later, buckling optimization of composite structures was considered in a finite element framework where the buckling load was determined by the solution to the linearized discretized matrix eigenvalue problem at an initial prebuckling point. Optimization of laminated composite plates has been studied by [27–32], while others considered more complex composite structures as curved shell panels and circular cylindrical shells, see [33–40]. However, applications of optimization methods to stability analysis and design of a general type of complex laminated composite shell structures have been very limited. To the best knowledge of the authors only one paper reports on nonlinear gradient-based buckling optimization of composite laminated plates and shells, namely the paper by [41], where limit load optimization is considered.

Another important topic in structural stability is the study of the influence of initial imperfections. Imperfections are deviations from the perfect structure, i.e. the analysis model, and can in general be geometrical, structural, material or load related. Despite that initial imperfections may be important in terms of the stability load of a structure it is not considered in the present paper.

This paper presents an integrated and reliable method for doing optimization of composite structures w.r.t. stability by including the nonlinear response by a path tracing analysis, here by the arc-length method, in the optimization formulation using the Total Lagrangian formulation. The nonlinear path tracing analysis is stopped when a limit point is encountered and the critical load is approximated at a precritical load step according to the "one-point" approach. Design sensitivities of the critical load factor are obtained semi-analytically by the direct differentiation approach on the approximate eigenvalue problem described by discretized finite element matrix equations. A number of the lowest buckling load factors are considered in the optimization formulation in order to avoid problems related to "mode switching" well-knowing that issues may be encountered due to divergence of the displacement sensitivities. The proposed method is benchmarked against a formulation based on linear buckling analysis on two engineering examples of laminated composite structures. This will help clarify the importance of the nonlinearity in structural design optimization w.r.t. stability.

In this work only Continuous Fiber Angle Optimization (CFAO) is considered thus fiber orientations in laminate layers with preselected thickness and material are chosen as design variables in the laminate optimization.

The "traditional" linear formulation for buckling analysis, sensitivity analysis and optimization formulation is outlined in Sections 2 and 3. In Section 4 the proposed procedure regarding nonlinear buckling analysis is stated. Derivations of design sensitivities, using the direct differentiation approach, of the nonlinear buckling load are presented along with the nonlinear buckling optimization formulation in Section 5. Both methods are benchmarked upon engineering examples of laminated composite structures. In Section 6 a laminated composite U-profile is studied while a much more complicated structure of a generic main spar of a wind turbine blade is studied in Section 7. Conclusions are outlined in Section 8.

2. Linear buckling analysis of laminated composite shell structures

The finite element method is used for determining the linear buckling load factor of the laminated composite structure, thus the derivations are given in a finite element context.

A laminated composite is typically composed of multiple materials and multiple layers, and the shell structures can in general be curved or doubly-curved. The materials used in this work are fiber-reinforced polymers, e.g. Glass or Carbon Fiber-Reinforced Polymers (GFRP/CFRP), oriented at a given angle θ_k for the kth layer or softer isotropic core material. All materials are assumed to behave linearly elastic and the structural behaviour of the laminate is described using an equivalent single layer theory where the layers are assumed to be perfectly bonded together such that displacements and strains will be continuous across the thickness.

The solid shell elements used for all the examples in this paper are derived using a continuum mechanics approach so the laminate is modelled with a geometric thickness in three dimensions, see [42]. The element used is an eight node isoparametric element where shear locking and trapezoidal locking are avoided by using the concepts of assumed natural strains (ANS) for, respectively, out-of-plane shear interpolation, see [43], and through-the-thickness interpolation, see [44]. Membrane and thickness locking is avoided by using the concepts of enhanced assumed strains (EAS) for the interpolation of the membrane and thickness strains, respectively, see [44] and [45]. The EAS interpolation is used to enhance the compatible strain tensor with an independent incompatible strain tensor, and the solid shell element used has seven internal degrees of freedom for the representation of the enhanced strains. This is the lowest number of internal degrees of freedom to introduce for the enhanced strains if the element should pass the in-plane membrane and out-of-plane bending patch tests, see [46] for details.

The static equilibrium equation for the structure may be written as

$$\mathbf{K}_0\mathbf{D} = \mathbf{R} \tag{1}$$

Here $\mathbf{D}$ is the global displacement vector, $\mathbf{K}_0$ is the global initial stiffness matrix, and $\mathbf{R}$ the global load vector.

Based on the displacement field, obtained by the solution to Eq. (1), the element layer stresses can be computed, whereby the stress stiffening effects due to mechanical loading can be evaluated by computing the initial stress stiffness matrix $\mathbf{K}_\sigma$. By assuming the structure to be perfect with no geometric imperfections, stresses are proportional to the loads, i.e. stress stiffness depends linearly on the load, displacements at the critical/buckling configuration are small, and the load is independent of the displacements, the linear buckling problem can be established as

$$\left(\mathbf{K}_0 + \lambda_j\mathbf{K}_\sigma\right)\mathbf{\Phi}_j = \mathbf{0},\ j = 1, 2, \ldots, J \tag{2}$$

where the eigenvalues are ordered by magnitude, such that λ_1 is the lowest eigenvalue, i.e. buckling load factor, and $\mathbf{\Phi}_1$ is the corresponding eigenvector i.e. buckling mode. In general, for engineering shell structures, the eigenvalue problem in Eq. (2) can be difficult to solve, due to the size of the matrices involved and large gaps between the distinct eigenvalues. For efficient and robust solutions, Eq. (2) is solved by a subspace method with automatic shifting strategy, Gram–Schmidt orthogonalization, and the sub-problem is solved by the Jacobi iterations method, see [47].