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A simple and robust three-dimensional cracking-particle method without enrichment

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ABSTRACT

A new robust and efficient approach for modeling discrete cracks in meshfree methods is described. The method is motivated by the cracking-particle method (Rabczuk T., Belytschko T., International Journal for Numerical Methods in Engineering, 2004) where the crack is modeled by a set of cracked segments. However, in contrast to the above mentioned paper, we do not introduce additional unknowns in the variational formulation to capture the displacement discontinuity. Instead, the crack is modeled by splitting particles located on opposite sides of the associated crack segments and we make use of the visibility method in order to describe the crack kinematics. We apply this method to several two- and three-dimensional problems in statics and dynamics and show through several numerical examples that the method does not show any "mesh" orientation bias.

1. Introduction

The simulation of a large set of evolving cracks by finite element or meshfree methods still poses substantial difficulties. Two classes of methods for such problems are typically used. The first class of methods enforces crack path continuity. Here we can classify methods into two further classes: methods that can have embedded cracks and methods that cannot have embedded cracks. Interelement separation methods ([23,43,69–71]) belong to the latter group. In interelement separation models, cracks are only allowed to develop along existing interelement edges. This endows the method with comparative simplicity, but can result in an overestimation of the fracture energy when the actual crack paths are not coincident with element edges. The results severely depend on the shape and mesh orientation of the mesh. This mesh dependence can only be alleviated by excessive remeshing that is computationally expensive.

Methods that are able to embed the discontinuity, i.e. the crack, in the element are embedded elements [1,12,39–41], the extended finite element method (XFEM) [8,19,21,22,36] or the generalized finite element method (GFEM) [65]. In embedded elements, the crack can propagate only one element at a time and the cracks open piecewise constant though novel approaches also allow the crack to open piecewise linear [34]. Embedded elements principally allow the crack path to be non-continuous. However, if crack path continuity is not enforced in embedded elements, it is well known that the results will depend on the mesh orientation bias [37].

In the extended finite element method, the crack can open linearly (in case of linear shape functions) and non-linearly (in case of higher order shape functions). This requires the enforcement of crack path continuity since the crack has to close at its tip. The crack is usually described by level sets though this is not mandatory. When many cracks occur and when problems such as bifurcating cracks and joining cracks are considered, methods that enforce crack path continuity become cumbersome. Tracing the crack is one of the most difficult tasks when crack path continuity is enforced, especially in 3D. One difficulty is to distinguish between crack propagation and crack initiation. Another difficulty in dynamic applications is to decide when and whether a crack branches or not.

Crack *propagation* typically occurs when cracking is detected at a material point with a certain radius around the crack front while crack *initiation* occurs if cracking is detected outside that given radius. While this is unproblematic in quasi static applications with few cracks, it can cause severe difficulties in dynamic applications with many cracks.

To consider branching cracks, let us consider exemplary Fig. 1a. First of all, crack branching within a single element, Fig. 1b, requires the design of special elements as described e.g. in Daux et al. [25] and Belytschko [17]. Secondly, to the best of our knowledge, only empirical models exist in order to decide whether a crack branches or not. These models have to be used with care. The authors for example have used in [51,55] the deviation in the crack orientation in front of the crack tip as criteria for crack branching. Though we do not present a solution to this question and basically adopt similar criteria,

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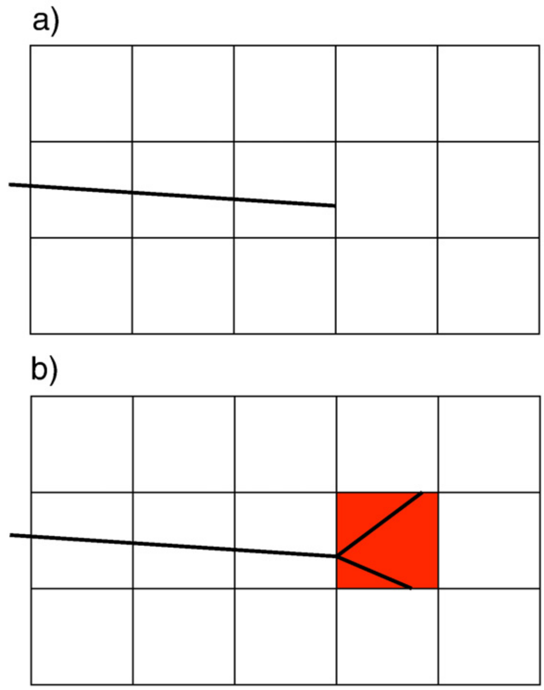

Fig. 1. a) Single crack that crosses several elements; b) crack branching (red element) that requires specially designed element formulations.

we avoid the problems of modeling the crack surface (explicitly or implicitly). One difficulty that occurs often in dynamic simulations is that the cracking criteria are often met at several sampling points in front of the crack tip simultaneously. It can also occur that the detected crack orientation of the new branches is not consistent with the old crack tip or the estimated crack speed. The problem gets even more complicated in three dimensions.

The situation in meshfree methods is similar. Most meshfree methods consider the crack as a continuous surface, [9,10,14,15,32]. Hence, the difficulties in meshfree methods are similar as compared to XFEM though no special treatment for "branched elements" has to be considered. However, the difficulty in deciding if crack branching or crack initiation occurs still remains. Usually, the cracking criteria in a certain radius around the crack tip are checked and if cracking is detected within this radius, the crack is assumed to branch, otherwise crack initiation is assumed.

Recently, methods have been proposed that do not enforce crack path continuity [26,47,58]. Though these methods are less accurate than methods that enforce crack path continuity, they are easier to implement and usually better suited for problems with excessive cracking since crack branching and crack junctions happen automatically. Also the problem of representing the crack surface can be avoided.

This paper is motivated by the *cracking-particle method* [47] where the crack was modeled by a discontinuous enrichment that can be arbitrarily aligned in the body at each particle (or node). The model of a continuous crack then consists of a set of contiguous cracked particles. In Rabczuk and Belytschko [48], it was shown that a discontinuous enrichment of cracked particles is not sufficient and particles in the blending domain were excluded in the approximation of the discontinuous displacement field. Within this paper, we follow the idea of modeling the crack as set of cracked particles. However, in contrast to the *cracking-particle method*, we do not introduce additional unknowns in the variational formulation to capture the displacement discontinuity. Instead, the particles, where cracking is detected, are split into two particles lying on opposite sides of the crack. To capture the jump in the displacement field, the shape functions of the particles adjacent to the cohesive crack segments are cut across the crack boundary similar to the visibility method, that was first proposed by Belytschko et al. [13], see also [14,15,42]. An effective implementation of the visibility method is given e.g. in Rabczuk and Belytschko [46]. The major advantage of the proposed method compared to the approach in [47] is that no additional unknowns need to be introduced. This allows the mass matrix to be diagonalized easily by a simple row-sum procedure (or other methods such as local reconstruction of the Voronoi cell). The method will be applied to several two- and three-dimensional problems in statics and dynamics. The method is of the same accuracy as the *cracking-particle method*. It will be shown that the method does not show any "mesh" orientation bias though no crack path continuity is imposed.

The article is arranged as follows: The governing equations and the element-free Galerkin (EFG) method are given in Sections 2 and 3, respectively. In Section 4, we explain the new cracking concept. The discrete linear momentum equation and the cracking criterion are given in Sections 5 and 6. In Section 7, we will present a cohesive law that takes dynamic effects in the fracture energy into account. In Section 8 we apply the method to several static and dynamic problems: a simple problem to examine locking effects, several prestressed concrete beams under four-point-bending, the Kalthoff problem, the fragmenting ring problem and concrete slabs under impact and explosive loading. Some of these results are compared to experimental data or other results from the literature.

2. Governing equations

Let us consider a body Ω in $\Re^3$ with boundary Γ; their images in the initial state are the open set Ω_0 and Γ_0, respectively. The strong form of the linear momentum equation in a Total Lagrangian description is:

$$\nabla_0 \cdot \mathbf{P} + \varrho_0 \mathbf{b} = \varrho_0 \ddot{\mathbf{u}} \text{ in } \Omega_0 \backslash \Gamma_0^c \tag{1}$$

where $\mathbf{P}$ is the nominal stress tensor (see [18] for details), $\mathbf{b}$ are the body forces, $\mathbf{X}$ are the material coordinates, ϱ_0 is the initial mass density, ∇_0 is the gradient operator with respect to the material coordinates, superimposed dots indicate material time derivatives or time derivatives depending on the context, Γ_0^c is the crack surface and Ω_0 is the domain of the body in the initial configuration. For static applications in Section 1, the inertia term vanishes. The boundary conditions are

$$\mathbf{n}_0 \cdot \mathbf{P} = \bar{\mathbf{t}}_0 \text{ on } \Gamma_0^t \tag{2}$$

$$\mathbf{u} = \bar{\mathbf{u}} \text{ on } \Gamma_0^u \tag{3}$$

$$\mathbf{n}_0^c \cdot \mathbf{P}^- = \mathbf{n}_0^c \cdot \mathbf{P}^+ = \mathbf{t}_0^c \text{ on } \Gamma_0^c \tag{4}$$

where $\bar{\mathbf{u}}$ and $\bar{\mathbf{t}}_0$ are the prescribed displacements and tractions, respectively, $\mathbf{t}_0^c$ are the cohesive forces across the crack, $\mathbf{n}_0^c$ is the normal to crack as shown in Fig. 2; $\Gamma_0 = \Gamma_0^c \cup \Gamma_0^t \cup \Gamma_0^u$ and $\Gamma_0^c \cap \Gamma_0^t \cap \Gamma_0^u = \varnothing$ where Γ_0^c is the crack surface.

3. The element-free Galerkin (EFG) method

We employ the EFG method, where an approximation in a Lagrangian description is given by

$$\mathbf{u}(\mathbf{X}, t) = \mathbf{p}^T(\mathbf{X})\mathbf{a}(\mathbf{X}, t) \tag{5}$$

where $\mathbf{X}$ are the material coordinates, t is the time and we have chosen $\mathbf{p}$ to consist of linear basis functions $\mathbf{p}(\mathbf{X}) = \{1\ X\ Y\ Z\} \forall \mathbf{X} \in \Re^3$. Minimizing

$$J = \sum_{I \in S} \left(\mathbf{p}_I^T(\mathbf{X})\mathbf{a}(\mathbf{X}, t) - \mathbf{u}_I(t)\right)^2 W(\mathbf{X} - \mathbf{X}_I, h) \tag{6}$$