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Three dimensional experimental and numerical multiscale analysis of a fatigue crack

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ABSTRACT

A full three dimensional study of a fatigue crack in cast iron is presented. This analysis involves combining various tools, namely, Synchrotron X-ray microtomography of an *in situ* experiment, image acquisition and treatment, 3D volume correlation to measure 3D displacement fields, extraction of the crack geometry, eXtended Digital Volume Correlation to account for the crack displacement discontinuity, crack modeling in an elastic material exploiting the actual crack geometry, and finally estimation of stress intensity factors. All these different tasks are based on specific multiscale approaches.

1. Introduction

To analyze the long-term reliability of structures, there is a need for robust and validated 3D crack propagation models and numerical procedures. This effort involves a number of specific challenges on the numerical modeling side because of the very demanding cost of three dimensional approaches. Validation itself appears to be even more challenging as it requires a full three dimensional comparison between models and experiments, and the latter requires three dimensional investigation instruments. The present study illustrates one of the routes that can be followed to achieve a direct link between experimental and computational mechanics [1–5].

Fatigue cracks are three dimensional objects in essence, but the lack of 3D data about crack propagation in optically opaque materials has made necessary the development of simplified 2D fatigue models. However, even for one of the simplest (and widely used) crack geometry like a through crack in a plate specimen, 2D measurements via an optical microscope only give access to the crack tip position on a free surface, *i.e.*, schematically in plane stress conditions while the crack propagates faster inside the specimen, *i.e.*, closer to plane strain conditions. The phenomenon of crack closure [6] can be used to account for the differences between surface and bulk propagation. It is usually assessed by considering surface observations, for instance by using displacement fields [7,8]. However, it was shown that this effect is mainly active in the vicinity of free surfaces [9]. This last result was obtained by coupling X-ray microtomography with X-FEM simulations relying upon a visual determination of the crack front.

In the present work, it is proposed to integrate even more the experimental analysis and the numerical tools by combining the following procedures (Fig. 1):

- X-ray tomography to get 3D pictures of *in situ* tests in, say, a synchrotron facility. By post-processing them, one may get, for instance, a first estimate of the morphology of the cracked surface,

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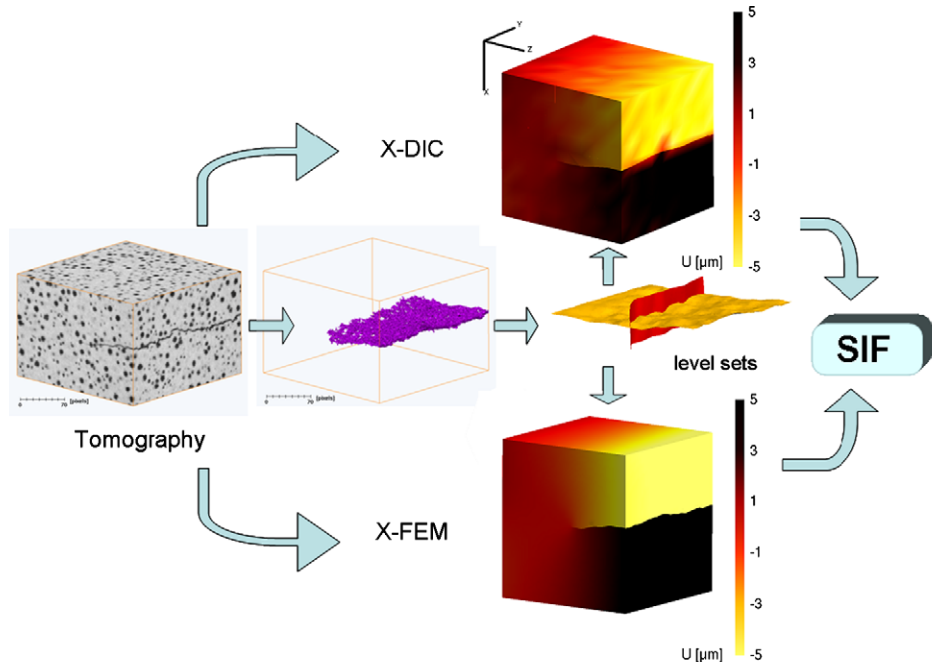

Fig. 1. Schematic view of the methods used in the present paper for determining stress intensity factors along the front of a crack imaged by computed tomography.

- volume correlation to measure displacement fields whose kinematic basis is consistent with experiments on cracked samples and subsequent X-FEM analyses,
- description of the crack shape by a level set modeling,
- multi-grid X-FEM simulation based on the actual crack shape determination,
- extraction procedures to identify stress intensity factors, either from the correlation (*i.e.,* measurement) procedure or from an elastic modeling.

X-Ray Computed MicroTomography (XCMT) is a very powerful way of imaging material microstructures in a non-destructive manner. By reconstruction from 2D pictures (radiographs), it allows for a 3D visualization of various phases of materials [10,11]. One has also access, for instance, to microstructural changes during solidification of alloys [12], to microstructural details during sintering of steel powders [13,14], to damage mechanisms in the bulk of particulate composites [15,16] or to the structure of cellular materials (either metallic or polymeric foams) and strains [17,18]. These actual microstructures may be further processed using finite element tools [19].

In the following, Digital Volume Correlation (DVC) and tomographic techniques are applied to measure displacement fields in nodular graphite cast iron when cracked (Section 2). This step is achieved by resorting to correlation techniques. DVC is a technique that consists in measuring displacement fields based on image pairs of the same specimen at different stages of loading [20]. The displacement field is computed so that the image of the loaded sample is matched to the reference image when voxel locations are corrected for by the displacement field. The most commonly used correlation algorithms consist in matching *locally* small zones of interest in a sequence of pictures to determine local displacement components [21]. The same type of hypotheses are made in three dimensional algorithms [22–25]. This study reports on a different algorithm, namely a Galerkin approach to Digital Image Correlation (DIC) that has been shown to be more performing in two dimensions when compared to traditional DIC algorithms [26,27]. It has been extended to three dimensions for continuous displacements [28]. Strain localization could be analyzed in a solid foam. Section 3 presents the application of this DVC algorithm to tomographic images.

The next step is to further extend the finite element shape function description of the kinematics to an enriched basis as used in X-FEM simulations [32] and previously performed to analyze 2D cases [33,34]. The enriched kinematics, in the spirit of X-FEM, needs a precise description of the crack geometry. Image analysis techniques (*e.g.,* binarization, skeletonization, extraction of connected component) have been successfully used for rather homogeneous materials [9]. In the present case, the heterogeneous microstructure, which is very helpful in terms of image correlation, becomes a strong obstacle to resort to such tools on 3D images. However, exploiting correlation residuals offers a very valuable field from which the crack surface may be extracted. A direct processing of this field, and a new automatic extraction algorithm are presented in Section 4.

Section 5 is devoted to the presentation of the eXtended DVC analysis, as previously performed to analyze 2D cases [33,34] and here extended to the three dimensional case. After introducing the theoretical framework, *a priori* performances are studied both to choose the appropriate size of the elements and to provide a comparison with non-eXtended DVC. Last, the displacement field obtained from the above technique based on the actual crack surface geometry is presented.

In a third step, the combination of XCMT and Digital Volume Correlation is completed with X-FEM simulations to improve the understanding of 3D crack growth laws. This numerical model is the combination of an external finite difference mesh (to describe the crack shape in the area of interest), an X-FEM mesh (*i.e.,* crack discontinuity and asymptotic behavior close to the crack front), a local multi-grid strategy to capture the possible scale effects with a high efficiency in terms of CPU time and numerical optimization (Section 6).

From the displacement field obtained by the previous analyses, it is possible to determine stress intensity factors. In 2D cases, the first route is a least squares minimization between the measured displacement field and its analytic asymptotic development around the crack tip [35]. The same type of fields can be implemented in an integrated DIC (I-DIC) algorithm [36,8]. A second