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A multi-level wave based numerical modelling framework for the steady-state dynamic analysis of bounded Helmholtz problems with multiple inclusions

Bert Van Genechten *, Karel Vergote, Dirk Vandepitte, Wim Desmet

K.U. Leuven - Dept. of Mechanical Engineering, Celestijnenlaan 300B - box 2420, 3001 Heverlee (Leuven), Belgium

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ABSTRACT

The Wave Based Method (WBM) is an alternative numerical prediction method for both interior and exterior steady-state dynamic problems, which is based on an indirect Trefftz approach. It applies wave functions, which are exact solutions of the governing differential equation, to describe the dynamic field variables. The smaller system of equations and the absence of pollution errors make the WBM very suitable for the treatment of Helmholtz problems in the mid-frequency range, where element-based methods are no longer feasible due to the associated computational costs. A sufficient condition for convergence of the method is the convexity of the considered problem domain. As a result, only problems of moderate geometrical complexity can be considered and some geometrical features cannot be handled at all. In this paper, these limitations are alleviated through the development of a general modelling framework based on existing WBM methodologies which allows for the efficient introduction of inclusion configurations in bounded WBM models for problems governed by one or more Helmholtz equations. The feasibility and efficiency of the method is illustrated by means of numerical verification studies in which the methodology is applied to two types of dynamic problems. On the one hand, a single Helmholtz equation associated with the steady-state dynamic behaviour of acoustic cavities is studied. On the other hand, the framework is applied to the solution of the Navier system of partial differential equations that describe the elastodynamic response of two-dimensional perforated solids.

1. Introduction

In recent years, the application of numerical simulation techniques in design, analysis and optimisation of mechanical systems has become an indispensable part of the industrial design process. Both the Finite Element Method (FEM) and the Boundary ElementMethod (BEM) are well established Computer Aided Engineering (CAE) tools which are commonly used for the analysis of time-harmonic dynamic problems.

The FEM [1] discretises the problem domain into a large but finite number of small elements. Within these elements, the dynamic field variables are described in terms of simple, polynomial shape functions. However, since these shape functions are no exact solutions of the governing differential equations, a fine discretisation is required to suppress the associated pollution error [2] and to obtain reasonable prediction accuracy at higher frequencies. Solving the resulting large numerical models requires a prohibitively large amount of computational resources. As a result, the FEM is limited to low-frequency applications [3].

In recent years, a vast amount of research has been done into the development of possible extensions of the FEM in order to minimise or even eliminate the numerical pollution effects and, as a result, increase the practical application range of the method to higher frequencies. This has lead to a wide range of techniques, which can be classified into a number of categories based on their specific focus. A first family of approaches attempts to optimise the FE modelling process without fundamentally altering it. Among these, refinement methods aim at reducing the approximation errors through adaptive (local) reduction of the dimensions of the FE discretisation [2] or through an elevation of the approximation order of the FEM basis functions [4] or through a combination of both [5]. Alternatively, [6–9] propose special numerical integration schemes which significantly increase the accuracy of the FEM with respect to numerical pollution errors. Advanced iterative solution strategies can also be employed to solve the FE numerical models more efficiently [10,11]. The main drawback of these solution algorithms is that they are less robust and that the gain in computational efficiency is highly dependent on the problem at hand and the combination of iterative solver and preconditioner used to solve the numerical systems. Domain decomposition techniques, like Component Mode Synthesis [12], its automated iterative variation Automated Multi-Level Substructuring [13,14] and the Finite Element Tearing and Interconnecting approach [15,16], apply a *divide and conquer* strategy which is perfectly suited

* Corresponding author. Tel.: +32 16 32 86 06; fax: +32 16 32 29 87.
E-mail address: Bert.VanGenechten@mech.kuleuven.be (B. Van Genechten).

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for parallel implementation. A final family of FEM extensions modifies the underlying integral formulations. Examples of this approach are the stabilised FE methods [17–19], the ultra-weak variational formulation [20,21] and multi-scale methods like the discontinuous enrichment [22–24] and discontinuous Galerkin techniques [25,26]. Although all these developments have been instrumental towards alleviating the frequency limitations of the FEM, they share the property that the solution strategies are based on a discretisation of the problem domain which has to conform to the boundary geometry of any inclusion present in the problem domain, like e.g. holes, voids, particles or aggregates. Even though at present many mesh generation algorithms are well established, the creation of high-quality FEM discretisations of domains with an arbitrary number and distribution of defects and inclusions remains a challenging and time-consuming process. Moreover, the main motivation for the development of many of these techniques is to permit relatively rough element sizes to be used up to much higher frequencies at the cost of more complicated, and computationally more expensive, element formulations. When problems with complex multiple inclusion configurations are considered, the gain in computational efficiency of these methods is partly negated by the need to use conforming meshes since in this case the required element sizes are governed by the need to accurately capture the problem geometry rather than by the frequency limitations of the method.

In order to overcome such meshing problems, several special techniques have been developed, which can be divided into two groups. On the one hand there are the techniques which contain special elements of which the problem boundary does not have to coincide with element boundaries. Examples are the extended FEM and the generalised FEM [27–29], in which the finite element approximation fields in elements near the inhomogeneities are enriched (in a partition of unity way) using fields with a strong physical meaning related to the properties of the inhomogeneities [30,31], or techniques which introduce problem-dependent specialist elements in the vicinity of the the inhomogeneity [32–34] while classical finite elements are used to model the remainder of the problem domain. These elements typically use analytical solutions of the governing equation to represent the local behaviour near the inhomogeneity. These approaches have, to the author's knowledge, only been applied successfully to static problems. Due to the mesh resolution requirements resulting from the study of dynamic problems at higher frequencies, the relatively large specialist elements, which typically enclose an entire hole, particle, ..., will couple more and more finite element degrees of freedom to each other, resulting in a significant increase in the bandwidth of the FEM system matrix and a drastic decrease in modelling efficiency. On the other hand, there are techniques which offer elements with a great geometrical flexibility, for example the NURBS-based isogeometric analysis developed by Hughes et al. [35]. However, research on that method has, in the current field of interest, mainly focussed on static problems and free vibrations of structures [36]. A rigorous assessment of the behaviour of this method for the forced response analysis of mid-frequency problems containing multiple inhomogeneities has to the author's knowledge not been performed at present.

In contrast to the previously described FEM and FEM-based techniques, which discretise the entire problem domain into small elements, the BEM [37] is based on a boundary integral formulation of the problem, such that only the boundary of the considered domain has to be discretised. Within the applied boundary elements, some boundary variables are expressed in terms of simple, polynomial shape functions. Enforcement of the boundary conditions results in a small numerical model, as compared to FE models, which can be solved for the nodal values at the discretised boundary. Once these nodal values are known, the field variables inside the domain may be reconstructed by application of the boundary integration formulations in a post processing step. While the use of a boundary discretisation eliminates the problems faced by domain discretisation methods for problems with complex inclusion configurations, the construction of the frequency-dependent, complex, densely populated BE matrices, which includes the integration of singular functions, is very time consuming as compared to the fast assembly of frequency-independent, real valued, sparse FE matrices. In this way, the smaller model size does not necessarily result in an enhanced computational efficiency, so that the practical use of the BEM is also restricted to low-frequency applications [38]. Moreover, when the complexity of the inclusion geometries increases, the number of boundary values grows, resulting in a further lowering of the practical application range of the method.

Apart from the FEM and BEM and all the methods derived from their basic concepts, there is another family of methods, the so called Trefftz methods [39], which distinguish themselves from the FEMs by their choice of shape and weighting functions [40]. Instead of using approximation functions, exact solutions of the governing differential equations are used for the expansion of the field variables. One such Trefftz based method is the Wave Based Method (WBM) [41]. It is a novel numerical prediction method for the analysis of steady-state interior and exterior Helmholtz problems. Since the functions which are applied to expand the dynamic field variables are exact solutions of the governing (system of) Helmholtz equation(s), no residual error is involved with the governing partial differential equation inside the problem domain. However, the functions may violate the boundary conditions. Enforcing the residual boundary errors to zero in a weighted residual scheme yields a small matrix equation. Due to the small model size and the enhanced convergence characteristics of the WBM, it has a superior numerical performance as compared to classical domain discretisation methods. As a result, problems at higher frequencies can be addressed. In the past, the WBM has been successfully applied for the analysis of interior and exterior (vibro-) acoustic problems [42–45] and for the structural dynamic analysis of flat plates [46,47].

One of the important disadvantages of the WBM is the requirement of a division of the problem into convex subdomains to ensure convergence. This requirement necessitates a complex domain splitting for some problem geometries, while other geometries may not be possible at all, e.g. circular holes. Recent work by the authors [48,49] proposes a significantly enhanced multi-level version of the WBM for the analysis of two-dimensional acoustic scattering analysis of a configuration of well separated generally shaped obstacles in an unbounded acoustic medium. In this publication, the basic principles of this approach are further developed into a general modelling framework for the efficient introduction of inclusion configurations in bounded WBM models. Moreover, the applicability of the methodology is extended to encompass any type of dynamic problem governed by one or multiple coupled Helmholtz-type partial differential equations which is illustrated by a numerical verification study of the technique for both steady-state acoustic and elastodynamic problems. The proposed methodology is based on a decomposition of the original (bounded) problem into a single interior subproblem and one or multiple exterior problems. The outer boundary of the interior subproblem matches the outer boundary of the original problem domain while disregarding the inclusions. Each of the exterior problems describes the scattering behaviour of a single inclusion which is embedded in an unbounded homogeneous domain. By modelling each of the subproblems using existing WBM modelling methodologies and applying the superposition principle to the resulting response fields, an efficient and flexible numerical strategy for modelling mid-frequency dynamic multiple inclusion problems is obtained. Concerning the modelling principle, the proposed method is quite similar to the decomposition into two parts using the superposition principle in order to determine the stress concentration factor for an infinite plate with a hole under uniaxial tension, as proposed by Chen in [50]. In that paper, these principles are however only applied to the static analysis of an infinitely extended medium