Comput. Methods Appl. Mech. Engrg. 198 (2009) 1839–1847

Contents lists available at ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

An adaptive multi-scale computational modelling of Clare College Bridge

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ARTICLE INFO

Article history:
Received 22 February 2008
Received in revised form 25 September 2008
Accepted 31 December 2008
Available online 14 January 2009

Keywords:
Dynamic unilateral contact
Coulomb friction
Linear elasticity
Finite elements
Mathematical programming
Masonry structures

ABSTRACT

Masonry structures may be modelled as an assembly of linearly elastic bodies (individual bricks or stone-blocks) in unilateral frictional contact. Such models generally constitute a formidable computational challenge owing to the need to resolve interactions between individual bodies, such as detection of crack and openings and the resolution of non-linear equations governing the contact. Even for medium size structures, the large number of blocks from which they are assembled renders a full direct simulation of such structures practically impossible. In this paper, an adaptive multi-scale technique for the modelling of large-scale dynamic structures is implemented and applied to the computer simulation of Clare College Bridge, in Cambridge, UK. The adaptive multi-scale approach enables us to carry out simulations at a complexity normally associated with the cost of modelling the entire structure by a simple continuum model whilst incorporating small scale effects, such as openings of gaps and slippage between individual masonry units, using a systematic and locally optimal criterion.

1. Introduction

Masonry structures are often part of a nation's cultural heritage and a large amount of work has been dedicated in recent years to the development of reliable methods to assess their behaviour, strength, and stability, as well as the adequacy of techniques for their conservation and restoration (cf. e.g. [33,12,66,74,57]). However, more efforts are necessary if such methods are to be applied to practical structures, especially to those of very large-scale.

Large-scale structures made from bricks and stones exhibit a deceptively complex mechanical behaviour, due to the number and nature of their constituents (bricks or stone-blocks, and mortar) and the variety of loads (dead loads, live loads, differential settlements, earthquakes) that act upon them (see e.g. review article [54]). From an historical perspective, since the seventeenth century (e.g. Hooke 1675, Coulomb 1781, Young 1807, Navier 1826), both the linear-elastic and the plastic theories for the study of these structures have been developed (see [75,43,13–18,52]).

In this paper, we model the behaviour of a masonry structure under the assumptions that masonry has no significant tensile strength in the joints and that the crushing strength of the blocks is sufficiently high to ensure that these blocks do not break and all the plastic phenomena in masonry occur at the joints, cf. [41]. The deformation of the masonry units is taken into account by modelling the individual units as linearly elastic blocks bonded by potential fracture/slip lines at the joints (see also [37,59,69,60,53,48,34,1,70]). Alternative approaches treat the structure as a collection of rigid, non-deformable, blocks separated by joints (see [25–27,67,35,63,10]). The approach described here could also be applied to the study of a masonry structure modelled as a collection of rigid blocks by letting the value of the Young modulus go to infinity so that, in effect, the blocks are infinitely stiff.

For present purposes, we treat large-scale structures as being assembled from elastic blocks in mutual normal cohesive, frictional contact, where the relative normal contact is described by the Karush–Kuhn–Tucker or KKT conditions [50], while the frictional contact is modelled by the Coulomb dry friction law [24] (see also [42]). Moreover, we are concerned with a full dynamic multi-body contact problem modelling a masonry structure, rather than with a simplified quasi-static model (for a static frictionless model, see [3]). The dynamic model is more realistic and, from the mathematical view-point, renders itself to numerical approaches that can handle large variations in the input data in a more robust fashion. The complexity of this type of problem is commonly associated with the requirement for a unified treatment of the block interactions, involving the dynamical detection of contacts and openings and the resolution of non-linear conditions for contact, subject to the geometry, the boundary conditions, and the contact constraints.

Computational methods for contact/impact problems have been under development for more than two decades and the finite element approximation has found many applications in different

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doi:10.1016/j.cma.2008.12.030

areas of structural mechanics (see e.g. [68,19,6,28], and the monographs [47,49,32,2]). However, even for medium size masonry structures, the large number of blocks from which they are assembled makes a full detailed simulation of such structures practically impossible. Consequently, alternative macro-scale models have been proposed which treat masonry as an homogenised continuum. In this case also, the masonry continuum is considered as being assembled from either rigid particles (e.g. [61,20,76,21]) or linear-elastic blocks (e.g. [8,9,71,58,55,56,7,20,62,64,65]). On the one hand, such models allow for the distribution of stresses in a large-scale structure to be traced, but on the other hand, fail to adequately represent typical local damage and failure mechanisms such as cracking that may lead to changes in the topology and mass distribution, and which in turn can have significant consequences on the overall behaviour of the structure cf. [41]. Consequently, there is a need for models and methods capable of predicting the full behaviour of a structure, from initial linear-elastic deflections, to plastic failure mechanisms.

In [5], an adaptive multi-scale approach for the modelling of large-scale structures is proposed, which combines in a natural way data from the simplified structural level and the individual units level to retain in the computational process only those contact constraints which significantly contribute to the changing behaviour of the given structure. The approach involves the solution of a sequence of mathematical programming problems to reproduce the physical behaviour of the elastic multi-body contact problem in the dynamic structure. By using a reduced number of constraints, the multi-scale approach retains the fidelity of the model, in which every individual masonry unit is included, at a small fraction of the computational cost. In this paper, the adaptive multi-scale approach introduced in [5] is applied to the simulation of an existing structure, namely Clare College Bridge, in Cambridge, UK (Fig. 1).

Clare College Bridge was built in 1638-40 of Ketton stone ashlar. This is the oldest surviving bridge over the river Cam, standing for nearly 350 years. It contains three arches separated by two river piers, and its dimensions are relative to the central span of 6.72 m and a circumferential length of each voussoir of 25 cm. Its severe deformation was caused at an early stage by the movements of a bank abutment and one of the river piers. The inclination of the western pier under the differential settlement of its foundation and further movements of the western abutment to the west have caused reduction in the horizontal thrust leading to deflection of the crown in the central arch and significant distortion of the western arch (Fig. 2), all of which are also visible at the road level over the bridge. The numerical results in this paper are based on the physical parameters given in [40] and correspond to the structure in its currently deformed state. In [40], the bridge is studied in the general context of plastic theory (see also [41]). In [23], both rigid and linearly elastic bodies are considered for the 2D simulation of Pont Julien, South of France, modelled by the non-smooth contact dynamics (NSCD) discrete element technique.


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Fig. 2. Clare College Bridge: western span and pier (north façade).

The content of the remaining sections of this paper is as follows: In Section 2, we discuss the adaptive multi-scale modelling technique in details suitable for its computer implementation. In order to simulate the specific behaviour of Clare College Bridge, it suffices to focus our attention on a reduced two-dimensional (2D) model. As in [40], our adopted 2D model is based on the assumption that the façades (20 cm thick) form self-contained structures capable of supporting the parapet. In Section 3, we present the computational results and comment on the performance of the adaptive multi-scale procedure by which these results have been derived. Concluding remarks are addressed in Section 4.

2. Adaptive multi-scale modelling strategy

For a masonry structure modelled as a dynamic system assembled from elastic blocks in normal cohesive contact with Coulomb friction, and subjected to volume and surface forces, the corresponding system of partial differential equations with contact constraints can be written as a hyperbolic quasi-variational inequality, i.e. a time-dependent variational problem with non-linear inequality constraints (see [31,36,47,49,38,32]). This is discretised in space by finite elements and in time by an implicit time-stepping scheme that is energy-consistent (i.e. the only dissipation of the total energy is due to the frictional slip, while for frictionless contact the total energy is conserved) cf. [22].

First, we discretise the continuous problem in space, by piecewise linear finite element method with nodal basis satisfying the usual requirements of consistency with the boundaries, such that the nodes lying on the interfaces belong to the triangulations of the blocks on both sides of the interface. For comprehensive details


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Fig. 1. Clare College Bridge (north façade).