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Advances on the domain decomposition solution of large scale porous media problems

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ABSTRACT

In this paper, a family of state-of-the-art parallel domain decomposition methods that combine the advantages of both direct and iterative solvers are investigated for the monolithic solution of the **u**–*p* formulation of the porous media problem. Moreover, a new family of parallel domain decomposition methods, specifically tailored for the above problem formulation is presented which outperforms the current state-of-the-art parallel domain decomposition solvers. The power of this family of solvers is demonstrated in two large scale porous media problems.

1. Introduction

For single-phase media, the prediction of the ultimate failure load of a soil mechanics problem is possible with a reasonable computational effort and relatively standard mathematical formulations. For porous media problems, which are usually dynamic, a more involved formulation is required for limit load predictions. This complication stems from the fact that the behavior of geomaterials, where the pores of the solid phase are filled with one or more fluids, cannot be described by the laws governing single-phase media. Multi-phase media is frequently encountered in geotechnical problems such as uneven settlements of underlying soil deposits, superstructure damage due to consolidation of the foundations, dissipation of excess pore water pressure resulting from foundation loading. Furthermore, multi-phase consideration is required in seismic or wave loading of soil–structure interaction problems and draining systems in landfills and reservoirs.

Soils and geomaterials, in general, have an internal pore structure that partially consists of a solid phase, often called the solid matrix, and a remaining part, called the void space which is filled by a single or a number of fluid phases, like gas, water, oil, etc. The solid phase and the fluid phase(s) have different motions; due to these motions and the different material properties of the solid and the fluid phase(s), the interaction between them is of crucial importance, thus making the description of the mechanical behavior of the porous media more complicated. Moreover, this solid–fluid interaction is particularly strong in dynamic loading and may lead to fatal strength reduction of the porous medium, like the one that occurs during liquefaction of loose saturated granular soils subjected to repeated cyclic loading or during the localized failure in earth dams and wet embankments.

Significant progress has been made in the last three decades, both from a theoretical as well as experimental point of view, as far as understanding the behavior of the solid phase interacting with one or more fluids inside a porous medium. This progress is greatly based on the proposition of the effective stress [1] which was widely accepted and has been successfully applied for studying time-dependent problems of this kind. More recently, advances in the application of Biot's theory to the mechanical response of porous media have been documented by various researchers [4,5] while the three most important theories (Biot [2,3], classical mixture and hybrid mixture theory [6–10]) were shown in [15] to lead to a similar form of field equations.

The finite element method has been proven to be an extremely powerful tool for the determination of both stresses and pore fluid pressure(s) distribution in porous media. The first application of the finite element method to solve Biot's field equations for the consolidation problem in the plane strain condition can be found in [16]. Furthermore, a numerical analysis of anisotropic consolidation of layered media is presented in [17], while in [18] Biot's theory is re-examined where a simplification, by ignoring the relative acceleration of pore fluid(s) with respect to the soil skeleton is

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suggested. This approach was followed by many researchers and constitutes the basic problem formulation in the present investigation. Recently, a number of works have been published in porous media problems dealing with adaptive refinement techniques [14], mortar finite elements discretization [13], a posteriori error estimation [12] and stochastic multiscale methods [11] among others.

There are various successful methods for the numerical solution of finite element uncoupled problems. Domain decomposition methods (DDM) constitute an important category of methods for the solution of a variety of problems in computational mechanics. Their performance, in both serial and parallel computer environments has been demonstrated in a number of papers over the last decade. They are basically classified as primal and dual DDM. The primal DDM (P-DDM) reach the solution by solving for the interface displacements after elimination of the internal degrees of freedom (dof) of the subdomains, while dual DDM (D-DDM) proceed with the computation of the Lagrange multipliers required to enforce compatibility between subdomains after the elimination of all the dof (internal and interface) of each subdomain. In the early 90s, an important D-DDM, the Finite Element Tearing and Interconnecting (FETI) method was introduced [31] and recently a family of P-DDM, namely the P-FETI methods were proposed [32,33]. Since their introduction, FETI, P-FETI DDM and several variants have gained importance and today are considered as highly efficient DDM [39–43].

However, for porous media problems, the optimum solution method is still to be found. Several methods have been investigated [19–21,38] but a fully satisfactory answer has not yet been found. The monolithic approach, where all field equations are solved simultaneously, is regarded as the most suitable one [22,23] for this type of coupled problems and will be adopted in the present investigation. Solutions on parallel computing environments have also been investigated [24–26] by implementing methods primarily based on frontal and multi-frontal solution techniques, which were found to be less attractive since the computational overhead due to parallelization is significant [23].

Application of the aforementioned DDM for the solution of porous media problems has not yet been investigated. Its straightforward implementation raises a series of issues with the most important ones being the non-symmetric formulation of the problem and the handling of the rigid body modes. In this work, a family of innovative parallel domain decomposition algorithms based on the monolithic approach are implemented while addressing both these issues. An elegant symmetrization procedure of the implied dynamic equilibrium is proposed while primal and dual domain decomposition methods are implemented with special artificial coarse problems needed to remove the zero energy modes in the subdomains. Their performance will be demonstrated with numerical results of large scale porous media problems.

2. Porous media problem formulation

The basic equation that relates effective stresses, soil skeleton stresses and pore pressure for a multi-phase medium (Fig. 1) can be written as:

$$\boldsymbol{\sigma}'' = \boldsymbol{\sigma} + a\mathbf{m}^T p \tag{1}$$

with

$$\mathbf{m} = [1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0], \tag{2}$$

where $\boldsymbol{\sigma}''$ are the effective stresses, $\boldsymbol{\sigma}$ are the soil skeleton stresses and p is the pore pressure. The a coefficient takes values near unity for clay and sand soils and can be as low as 0.5 for rocky soils.

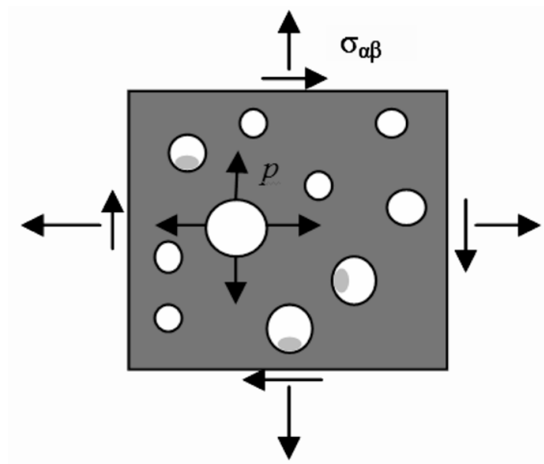

Fig. 1. A 2D porous material sample.

If p_w and p_a represent the fluid and air pore pressure respectively and χ_w and χ_a represent the percentage of the pore pressure, that is due to the existence of the fluid and the air in the pores, respectively, then in case of partially saturated media, where the air pressure is assumed to be negligible, the following equation holds:

$$p = \chi_w p_w + \chi_a p_a = \chi_w p_w + (1 - \chi_w) p_a \approx \chi_w p_w \tag{3}$$

with $\chi_w + \chi_a = 1$ and $\chi_w = \chi_w(S_w)$, where S_w is the media saturation degree.

Darcian flow **w** occurs when imposing a difference in hydrostatic pressure Δp between two material points, say A and B (Fig. 2).

If **R** is the viscous drag forces, **k** is the permeability matrix, having terms defined with dimensions of $[\text{length}]^3[\text{time}]/[\text{mass}]$ (and assuming isotropic permeability, $\mathbf{k} = k\mathbf{I}$ with k being the permeability coefficient), then $\mathbf{kR} = \mathbf{w}$.

Considering the soil skeleton and the fluid embraced by the usual control volume $dV = dxdydz$, the following equation describes the momentum balance relation for the soil–fluid mixture:

$$\mathbf{S}^T\boldsymbol{\sigma} - \rho(\ddot{\mathbf{u}} - \mathbf{b}) = 0 \tag{4}$$

with

$$\mathbf{S} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}^T \tag{5}$$

$\rho = nS_w\rho_w + (1 - n)\rho_s$ is the soil–fluid mixture density, ρ_s is the soil density, ρ_w is the fluid density, n is the porosity, **u** is the vector of the soil skeleton displacements, **b** is the vector of body force per unit mass and the dot refers to time differentiation. Convective terms regarding fluid flow have been omitted as they are generally very small and can be neglected [5].

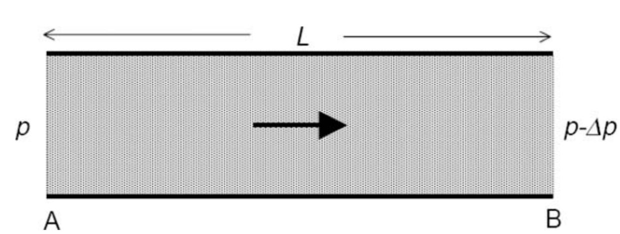

Fig. 2. Darcy flow through a medium under the action of a pressure gradient.