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Gradient and fracture energy-based plasticity theory for quasi-brittle materials like concrete

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ABSTRACT

In this work a thermodynamically consistent non-local gradient and fracture energy-based plasticity theory is proposed to simulate the failure behavior of concrete. The model incorporates two characteristic lengths, one due to the microcrack opening process and the other due to the non-local degradation process of the continuum in between cracks. The failure behavior of quasi-brittle materials like concrete is controlled by a decohesion mechanism expressed in terms of a combined fracture-energy and non-local gradient-based softening formulation. The two resulting characteristic lengths are functions of the stress state to describe the increasing non-locality of the degradation process as well as the reducing distance between microcracks with the increment of the confining pressure. In this way the transition from brittle to ductile post-peak response of quasi-brittle materials like concrete is realistically predicted. The thermodynamically consistent formulation covers both the hardening and softening regimes of the proposed constitutive model. The compressive meridian of the model maximum strength criterion agrees with that of Leon while constant and maximum value is adopted for the eccentricity leading to the circular forms of the failure surface deviatoric views similarly to the Drucker–Prager criterion. A volumetric non-associated flow rule is taken into account to appropriately describe the inelastic behavior of concrete in the low confinement regime. The predictive capabilities of the proposed constitutive formulation are tested against experimental results on concrete specimens in tensile and compressive regimes.

1. Introduction

It is well known that failure behavior of quasi-brittle materials like concrete does strongly depend on the governing stress and on the mechanical and chemical features of its micro and mesostructure. In the tensile regime the material response to mechanical loading is highly brittle as the damage entirely localizes in one single surface or crack of zero width. The failure cinematic is best described by means of the crack relative displacement and, consequently, the failure process and related mechanism exclusively depend on fracture energy properties. This assessment is valid for both normal and high strength concretes although relevant differences in the microcrack evolution mechanism as well as in the fracture energy features can be distinguished. In the case of normal strength concrete (*NSC*) the tensile or mode I failure process only affects the mortar. As a consequence, the crack follows a tortuous path due to the presence of the aggregates. In the case of high strength concrete (*HSC*) the superior mortar strength leads to an almost homogeneous distribution of the mechanical property between the components of its mesostructure. Consequently the tensile crack involves both mortar and aggregate leading to a more brittle failure process as compare to *NSC*. Despite these differences there is no doubt that the failure mechanism in the tensile regime of both *NSC* and *HSC* is fully controlled by the fracture energy release process in one single crack while the material outside the crack remains practically undamaged.

In the compressive regime the ductility of concrete failure behavior strongly increases with the confining pressure. The failure mechanism is characterized by both the appearance of several microcracks in the normal direction to the local maximum principal stress and the material damage process in the zones located in between cracks or microcracks. Under increasing monotonic loading both the microcracks and the portion of the material subjected to the degradation process coalesce in shear bands of pressure and aggregate size-dependent widths. In uniaxial compression the shear band width approximates the maximum aggregate size and continuously enlarges with increasing confinement levels. In other words, the failure behavior of concrete in the compressive regime is governed by the fracture energy releases in the microcracks as well as by the material degradation in between cracks. Moreover,

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the width of the localization process increases with the acting confining pressure.

This highly variable failure behavior of quasi-brittle materials like concrete is demonstrated by several experimental researches published in the literature, see a.o. [12,15,22,28]. The different failure mechanism of concrete in traction, uniaxial and triaxial compression tests can be recognized not only in the stress–strain responses but also in the evaluation of the cylindrical probes at final stage. In case of tests under uniaxial or biaxial traction the volume change is almost zero. However, in uniaxial compression and, particularly triaxial compression with increasing confining pressure, the volume change turns relevant due to the more comprehensive distribution of microcracks and to the enlargement of the damaged zone. Detailed descriptions on the mechanisms governing concrete strength degradation can be found in [21,29,31,38].

To accurately reproduce within one single formulation the entire spectrum of possible concrete failure modes when subjected to tensile or triaxial compression loading under low and high confinement levels and, moreover, the continuous transition from brittle to ductile failure behavior different theoretical frameworks need to be combined.

In the realm of the smeared crack approach, brittle materials require dissipative formulations based on fracture mechanics concepts. However, the simulation of mechanical degradation processes of ductile materials require non-local theories that appropriately and objectively describe the development of shear bands of non-zero thickness during loading processes beyond their limit strengths. Among the different non-local constitutive theories proposed in the literature for ductile materials, one of the most effective and convenient from the computational and calibration stand points is the strain gradient theory. Up to the present these two different approaches for constitutive model formulations (non-local gradient theory and fracture energy-based formulations) were alternatively considered, in spite of the fact that quasi-brittle materials like concrete may have brittle and ductile failure modes. Fracture energy-based concrete models are, among others, as described in [2–4,7,8,16,23,35,37]. Gradient-based constitutive models for cohesive materials like concrete and soils are, among others, as proposed in [5,6,18–20,25,30].

In this work a thermodynamically consistent material model for concrete failure behavior is proposed, the so-called Leon–Drucker–Prager (LDP) model, which is based on non-local gradient and fracture energy concepts and includes an isotropic hardening and softening formulation. In the hardening regime the material model if fully local. To account for the increasing ductility with the confinement that takes place in concrete pre-peak regime the hardening rule is formulated in terms of a pressure-dependent ductility measure. The strength degradation process in the post-peak regime is controlled by two independent mechanism. On the one hand, the decohesion due to the micro o macrocracking process and, on the other hand, the decohesion due to the degradation of the continuum or material located in between cracks. The first one is described with a fracture energy-based plasticity formulation similarly to the proposal by Willam et al. [35] and Etse and Willam [8]. In the case of mode I type of failure the fracture energy release is due to the opening process of one single crack and the fracture energy characteristic length representing the crack separation, takes its maximum value. In the case of mode II type of failure under high confinement, the fracture energy-based characteristic length reduces to its minimum value while the energy released during the microcrack opening process reaches its maximum possible value. The second decohesion process is modelled by means of gradient-based non-local plasticity with pressure-dependent characteristic length in order to appropriately predict the variable shear band width with the acting confinement.

The compressive meridian of the maximum strength surface of the proposed model agrees with the parabolic maximum strength criterion by Leon [13,14], while the Drucker–Prager circular description is adopted for its deviatoric view. Thus, a quadratic function for the failure surface in terms of two stress invariants is considered with constant eccentricity $e = 1$. This is a relevant advantage of the considered failure surface as it strongly facilitates the related numerical implementation. Nevertheless, the combined fracture energy and gradient-dependent constitutive theory for concrete in this work can be straightforwardly extended to include more complex and accurate maximum strength criteria.

Firstly the thermodynamically consistent gradient theory of plasticity considered for the formulation of the model is reviewed. This is based on the original proposal by Svedberg and Runesson [26] that was extended for gradient elastoplastic continua by Vrech and Etse [32]. Then the thermodynamically consistent fracture energy and gradient-based model for concrete is presented as well as the calibration of the internal functions and parameters. Finally, the model numerical predictions of concrete failure behavior in tensile and compressive regimes are illustrated and discussed. The numerical results show good agreement with the experimental ones indicating the capabilities of the proposed constitutive theory to capture the different and complex failure processes of concrete under tensile and triaxial compressive loading.

2. Thermodynamically consistent gradient plasticity

The present formulation follows Svedberg and Runesson [26]. According to Simo and Miehe [24] it is assumed that arbitrary thermodynamic states of the dissipative material during isothermal processes are fully defined by the elastic strains and a finite set of hardening/softening plastic variables. It is further assumed that there is only one internal (scalar) variable which is the only one of non-local character. Thus, the Helmholtz free energy density can be additively expressed as

$$\rho\Psi(\boldsymbol{\varepsilon}^e, \kappa, \nabla\kappa) = \rho\Psi^e(\boldsymbol{\varepsilon}^e) + \rho\Psi^p(\kappa) + \rho\Psi^g(\nabla\kappa), \tag{1}$$

where ρ is the material density. The elastic free energy is defined as

$$\rho\Psi^e(\boldsymbol{\varepsilon}^e) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \boldsymbol{E}^e : \boldsymbol{\varepsilon}^e, \quad \boldsymbol{E}^e = \frac{\partial^2\Psi^e}{\partial(\boldsymbol{\varepsilon}^e)\otimes(\boldsymbol{\varepsilon}^e)} \tag{2}$$

being $\boldsymbol{\varepsilon}^e$ and $\boldsymbol{E}^e$ the elastic strain tensor and the fourth-order elastic operator, respectively. The local and gradient free energy density contributions due to inelastic strains Ψ^p and Ψ^g, are expressed in terms of the scalar plastic variable κ. The gradient effects are only restricted to the hardening/softening behavior via the inclusion of $\nabla\kappa$.

From the Coleman's relations follows the constitutive equation for the stress tensor

$$\boldsymbol{\sigma} = \rho\frac{\partial\Psi}{\partial\boldsymbol{\varepsilon}}, \quad \boldsymbol{\sigma} = \boldsymbol{E}^e : \boldsymbol{\varepsilon}^e \tag{3}$$

with the dissipation within the continuum D and on the boundary D^b as

$$D = \boldsymbol{\sigma}\,\dot{\boldsymbol{\varepsilon}}^p + K\dot{\kappa} \geqslant 0, \quad \boldsymbol{x} \in \Omega, \tag{4}$$

$$D^b = K^b\dot{\kappa} \geqslant 0, \quad \boldsymbol{x} \in \partial\Omega, \tag{5}$$

whereby $\boldsymbol{\varepsilon}$ and $\boldsymbol{\varepsilon}^p$ are the strain and plastic strain tensors, respectively, while the "internal" dissipative stress is defined as, see [32]

$$K = K^p + K^g \tag{6}$$

being

$$K^p = -\rho\frac{\partial\Psi^p}{\partial\kappa} \quad \text{and} \quad K^g = \nabla\cdot\left(\rho\frac{\partial\Psi^g}{\partial(\nabla\kappa)}\right). \tag{7}$$