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Reliability-based optimization of stochastic systems using line search

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ABSTRACT

This contribution presents an approach for solving reliability-based optimization problems involving structural systems under stochastic loading. The associated reliability problems to be solved during the optimization process are high-dimensional (1000 or more random variables). A standard gradient-based algorithm with line search is used in this work. Subset simulation is adopted for the purpose of estimating the corresponding failure probabilities. The gradients of the failure probability functions are estimated by an approach based on the local behavior of the performance functions that define the failure domains. Numerical results show that only a moderate number of reliability estimates has to be performed during the entire design process. Two numerical examples showing the effectiveness of the approach reported herein are presented.

1. Introduction

Structural optimization is concerned with achieving an optimal design while satisfying certain constraints. In this regard, the optimal design can be defined as the best feasible design according to a preselected quantitative measure of effectiveness [3,10,11,14]. In most structural engineering applications, response predictions are based on structural models whose parameters are uncertain. This is due to a lack of information about the value of system parameters external to the structure, such as environmental loads (wind loading, water wave excitation, traffic loading, earthquake excitation, etc.) or internal such as material properties, construction defects and system behavior. Under uncertain conditions the field of reliability-based optimization provides a realistic and rational framework for structural optimization which explicitly accounts for the uncertainties [12]. It is noted that due to uncertain conditions, reliability-based optimization formulations are considerably more involved than their deterministic counterpart.

Reliability-based optimization requires advanced and efficient tools for structural modeling, reliability analysis and mathematical programming. Modeling and analysis techniques of mechanical systems based on local approximations are well established and sufficiently well documented in the literature [6,45]. On the other hand, several tools for assessing structural reliability have lately experienced a substantial development providing solution to a number of complex problems [5,7,19–21,23,31,38]. In the field of reliability-based optimization several procedures have been recently developed allowing the solution of quite demanding problems. Such procedures are usually based on a combination of approximation concepts with standard deterministic optimization techniques [1,8,9,13,16,17,22,26,29,30], or stochastic search algorithms [24,40,41]. The use of approximate models, i.e. meta-models, in reliability analysis and reliability-based optimization has been proposed in a number of publications [13,32,34,36]. In addition, recent developments of efficient and robust sensitivity analysis techniques are closely related to the construction of meta-models for complex structural systems [4,27,35,42,44]. Stochastic search algorithms have also proved to be useful tools for solving challenging optimization problems. In these approaches the values of the random functions are used directly as inputs to the optimization algorithm [40,41]. The algorithms used in these cases are generally direct search schemes which only use the values of random functions to be optimized as inputs. For a thorough review of the previous and other recent advances in the context of optimization problems considering uncertainties it is referred to, e.g. [37,39].

The use of the above optimization approaches has been found useful in a number of structural optimization applications. However, the application of reliability-based optimization to stochastic dynamical systems remains somewhat limited. For example, on one hand, meta-modeling techniques are not well suited to large scale optimization problems when the number of design variables is relatively large. This is specially prohibitive when considering large scale simulation models. On the other hand, most of the methodologies proposed in the literature for the solution of

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reliability-based optimization problems of stochastic dynamical systems do not possess proven convergence properties. Therefore, there is still much room for further developments in this area.

It is the objective of this work to implement a methodology for the solution of reliability-based optimization problems of stochastic dynamical systems with monotonic convergence properties. That is, the purpose of this paper is not in the development of new optimization algorithms but to introduce a general framework for solving a challenging class of structural optimization problems considering uncertainties. The solution of this type of problems is extremely demanding since involves reliability and sensitivity analyses in high-dimensional parameter spaces during the optimization process. Novel aspects of this contribution refer to an effective integration of the algorithm for optimization and reliability assessment. In particular, a new approach for efficient sensitivity estimation of probabilities is presented, which is based on an approximate representation of the structural response; this approach is numerically inexpensive, as it requires a single reliability analysis only, in addition to some structural analyses for estimating the sought sensitivities. The information on sensitivities is used in order to determine search directions in the space of the design variables within the optimization algorithm. In addition, a line search scheme specially designed for handling probabilistic constraints is introduced, in which a polynomial approximation of the probability is generated using information on the function value and its derivative (as suggested in [25,43]). The advantage of the proposed scheme is that it requires a very low number of reliability analyses in order to provide an accurate representation of the probability along the search direction.

The structure of this paper is as follows. In Section 2, the mathematical formulation of the reliability-based optimization problem is presented. Section 3 briefly describes the optimization algorithm used in this contribution (which is a well-known first-order scheme). Next, Section 4 addresses several implementation issues; among these, salient issues discussed are the approach for reliability sensitivity estimation and the application of the specialized line search scheme. Finally, two application problems are presented to illustrate the performance of the proposed methodology.

2. Reliability-based optimization problem

2.1. Formulation

Consider the following structural optimization problem:

$$\begin{aligned} \text{Min} \quad & C(\{x\}) \\ \text{subject to} \quad & h_i(\{x\}) \leqslant 0, \quad i=1,\ldots,n, \\ & g_i(\{x\}) \leqslant 0, \quad i=1,\ldots,m, \\ & s_i(\{x\}) = \{a_i\}^T\{x\} - b_i \leqslant 0, \quad i=1,\ldots,l, \end{aligned} \tag{1}$$

where $\{x\},\ x_i,\ i=1,\ldots,n_d$ is the vector of design variables, $C(\{x\})$ is the objective function, $h_i(\{x\}) \leqslant 0,\ i=1,\ldots,n$ are the reliability constraints, $g_i(\{x\}) \leqslant 0,\ i=1,\ldots,m$ are the deterministic non-linear constraints, and $s_i(\{x\}) \leqslant 0,\ i=1,\ldots,l$ are the deterministic linear constraints. The deterministic constraints are related to design requirements such as structural weight, geometric conditions and material cost components. The side constraints

$$\{x\} \in X, \quad x_i \in X_i = \{x_i | x_i^l \leqslant x_i \leqslant x_i^u\}, \quad i=1,\ldots,n_d \tag{2}$$

are included in the definition of the linear deterministic constraints $s_i(\cdot)$. The reliability constraints are written in terms of failure probability functions as

$$h_i(\{x\}) = P_{F_i}(\{x\}) - P^*_{F_i} \leqslant 0, \quad i=1,\ldots,n, \tag{3}$$

where $P_{F_i}(x)$ is the failure probability function for the failure event F_i evaluated at the design $\{x\}$, and $P^*_{F_i}$ is the target failure probability for the ith failure event. The failure probability function $P_{F_i}(\{x\})$ evaluated at the design $\{x\}$ can be written in terms of the probability integral

$$P_{F_i}(\{x\}) = \int_{\Omega_{F_i}} f(\{z\}/\{x\})\, d\{z\}, \tag{4}$$

where $\{z\} \in \Omega_{\{z\}} \subset R^{n_u}$ is the vector of uncertain variables involved in the problem, $f(\{z\}/\{x\})$ is the probability density function of $\{z\}$ conditioned on $\{x\}$, and Ω_{F_i} is the failure domain of failure event F_i in the $\Omega_{\{z\}}$ space. The failure domain Ω_{F_i} for a given design $\{x\}$ is defined in terms of the performance function κ_i as $\kappa_i(\{x\},\{z\}) \leqslant 0$, that is $\Omega_{F_i} = \{\{z\} | \kappa_i(\{x\},\{z\}) \leqslant 0\}$. Recall that $\{z\}$ is the vector of random variables that describes all uncertainties involved in the system (model and loading parameters). That is, the components of the vector $\{z\}$ represent the uncertain structural parameters and the random variables used in the characterization of the stochastic excitation. Therefore, the failure probability functions $P_{F_i}(\{x\}),\ i=1,\ldots,n$ account for the uncertainty in the system parameters as well as the uncertainties in the excitation. Finally, it is assumed that κ_i is a continuous function with respect to the design variables $\{x\}$.

2.2. Application to dynamical systems

For systems under stochastic excitation the probability that design conditions are satisfied within a particular reference period provides a useful reliability measure. Such measure is referred as the first excursion probability. In this case the failure events $F_i,\ i=1,\ldots,n$ are defined as

$$F_i(\{x\},\{z\}) = D_i(\{x\},\{z\}) > 1, \tag{5}$$

where

$$D_i(\{x\},\{z\}) = \max_{j=1,\ldots,n_j} \max_{t\in[0,T]} \frac{\left| r_j^i(t,\{x\},\{z\}) \right|}{r_j^{i^*}} \tag{6}$$

is the normalized demand, $[0,T]$ is the time interval, $r_j^i(t,\{x\},\{z\}),\ j=1,\ldots,n_j$ are the response functions associated with the failure event i, and $r_j^{i^*}$ is the corresponding critical threshold level. In this context the quotient $\left| r_j^i(t,\{x\},\{z\}) \right| / r_j^{i^*}$ is interpreted as a demand to capacity ratio, as it compares the value of the response $r_j^i(t,\{x\},\{z\})$ with the maximum allowable value of this response $r_j^{i^*}$. The response functions $r_j^i(t,\{x\},\{z\}),\ i=1,\ldots,n,\ j=1,\ldots,n_j$ are obtained from the solution of the equation of motion that characterizes the structural model. With the previous definition of normalized demand, the performance function can be written as

$$\kappa_i(\{x\},\{z\}) = 1 - D_i(\{x\},\{z\}) \tag{7}$$

and the corresponding first excursion probability can be expressed as the multidimensional integral

$$P_{F_i}(\{x\}) = \int_{\kappa_i(\{x\},\{z\})\leqslant 0} f(\{z\}/\{x\})\, d\{z\}. \tag{8}$$

It is noted that the normalized demand function and the performance function are in general non-smooth and therefore non-differentiable [18]. However, the differentiability of these functions is not required in the present formulation (see Section 4.2). It is also noted that the multidimensional probability integral (8) involves a large number of uncertain parameters (hundreds or thousands) in the context of dynamical systems under stochastic excitation. Therefore, Eq. (8) represents a high-dimensional reliability problem.