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Heuristic acceleration correction algorithm for use in SPH computations in impact mechanics

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ABSTRACT

Despite developments over the past 30 years, SPH and other mesh-free computational methods are not yet in general use as standard tools in dynamic structural mechanics. One possible reason for this is the use of features such as artificial viscosity, to stabilize the numerical computations, which can result in physically unreal phenomena. The effect of artificial viscosity in SPH computations is examined and a heuristic acceleration correction algorithm is proposed in this paper. The purpose is to improve the modelling of physically real effects and thereby make SPH a more attractive modelling option, particularly for structural impact problems.

The essence of the proposed method is to calculate the change in the acceleration due to the artificial viscosity term and then correct the computed acceleration by subtracting a kernel approximation of its artificial counterpart. The energy equation is also modified accordingly. By this means, the excessive dissipation is removed, while retaining the computational stabilizing effect of the artificial viscosity. For illustrative purposes, the proposed method is applied to several classical elastic and elastic–plastic impact problems and the results are compared with those available in the literature. In the process, the improved performance of the proposed algorithm vis-à-vis the standard SPH procedures is discussed as are the outstanding mathematical issues which require resolution to make the approach truly rigorous.

1. Introduction

Smooth Particle Hydrodynamics [16,6,18,20] is one of the oldest and probably the simplest of mesh-free methods. It is a grid-less Lagrangian technique, developed initially to deal with mathematical models of astrophysical and cosmological phenomena. SPH has been successfully applied to a wide range of problems in computational fluid dynamics [3] and has been extended to problems in solid mechanics by Libersky and Petschek [14] and Libersky et al. [15]. In demonstrating the value of the acceleration algorithm herein, we have followed the developments of Libersky and Petschek [14] closely.

Recently SPH has found considerable appeal amongst researchers for the numerical modelling of high velocity impact and penetration problems (e.g. [10,17]). Due to its particle nature, SPH is particularly very effective for modelling fragmentation and material separation caused by the formation of cracks and crack systems and the coalescence of a multitude of small crack-like flaws [1], which are major physical phenomena encountered in several areas of impact mechanics. These are not dealt with easily by other methods, e.g. FEM.

However, the method is not yet in general use as a standard tool in dynamic structural mechanics due to some inherent computational difficulties. These include the use of the artificial viscosity [22] to promote numerical stability, perhaps one of the major problems. In artificial viscosity formulations, an 'artificial pressure' term is added into the momentum and energy equations whenever the system experiences any shock compression. The most basic physical property that artificial viscosity generates is dissipation, i.e. it converts kinetic energy to internal energy. However, one has to be careful when choosing the artificial viscosity parameters that it does not induce any false pressure, which may lead to an excessive loss of kinetic energy, making the system over-dissipative and the predictions correspondingly physically unreal. These are of particular concern in impact mechanics problems.

Unfortunately, there is no standard procedure for choosing the artificial viscosity parameters which works for a range of problems. Most often, the SPH computations are performed with some generally arbitrarily-prescribed values of these parameters. Johnson [11] examined the effect of artificial viscosity in impact computations and showed that the accuracy of SPH computations can be significantly affected by the choice of artificial viscosity parameters. One way out of this difficulty is to perform numerical experiments (running the simulation with different values of the artificial viscosity parameters) in order to find the 'optimum' values of these

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parameters for a given problem. This is a formidable task when the size of the problem is large. Therefore an algorithm is needed which can bypass the requirement of user-defined parameters and yet controls the effect of artificial viscosity in order to prevent the system from being over dissipative, whilst still providing the necessary numerical stability.

In the present paper, an attempt has been made to accomplish the above objective by developing an *acceleration correction algorithm*. Herein, the acceleration equations are amended by subtracting a correction term. The correction term is taken as a kernel approximation of that part of the rate of momentum produced by the artificial viscosity, using the same kernel function by which the governing equations are discretized. The energy equation is also modified accordingly. For illustration, the proposed method is applied here to two classical elastic and elastic–plastic impact problems, the collinear impact of two elastic rods and a 2D representation of the Taylor bullet impact test. The results are compared with those available in the literature. The relative numerical advantages of the new algorithm are brought out in the process. A third classical problem in structural impact, the Parkes cantilever problem, is then given to illustrate its potential benefit for a wide range of structural impact problems.

The formulation of the acceleration correction algorithm presented below is heuristic, being based on intuition and numerical observations. Notwithstanding this, we have applied the algorithm to a series of problems related to impact mechanics and the results indeed are very encouraging. Although a detailed theoretical analysis of the proposed algorithm is yet to be produced, its numerical advantages vis-à-vis SPH models with standard artificial viscosity shows that the method has the potential to address the issues associated with using artificial viscosity in SPH computations.

The paper is organized as follows. In Section 2, the equations of SPH, as applied to elastic and elastic–plastic dynamics problems, are outlined. The notion of artificial viscosity is described in Section 3. The proposed correction algorithm is discussed in Section 4. The three test cases are provided in Section 5 to demonstrate the efficacy of the proposed method. As noted above, one of these is the Parkes cantilever problem [25], a classical, structural plasticity problem described in several textbooks and publications in the literature (e.g. [12,30,28,29,13]). Conclusions are drawn in Section 6.

2. SPH – A brief overview

In smooth particle hydrodynamics (SPH), first the entire domain is discretized by defining a set of particles. These particles interact with each other through a kernel function (sometimes also called the window or weight function) such that at every particle the conservation equations are satisfied. There exists extensive literature on SPH addressing different theoretical as well as numerical aspects [16,18,20]. The objective of this section is to outline the steps involved in SPH applied to solid mechanics problems so that a potential reader can transfer the theory into a computational code without further reference to other literature. However for more comprehensive information, readers are recommended to refer to the review paper by Monaghan [21] and the references therein.

2.1. Conservation equations

The conservation equations for continuum mechanics are,

$$\frac{d\rho}{dt} = -\rho\frac{\partial v^\beta}{\partial x^\beta}, \tag{1}$$

$$\frac{dv^\alpha}{dt} = -\frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta}, \tag{2}$$

$$\frac{de}{dt} = -\frac{\sigma^{\alpha\beta}}{\rho}\frac{\partial v^\alpha}{\partial x^\beta}, \quad \text{and} \tag{3}$$

$$\frac{dx^\alpha}{dt} = v^\alpha, \tag{4}$$

where, for any material point, ρ denotes its mass density, e is the specific internal energy, v^α and $\sigma^{\alpha\beta}$ are respectively the elements of velocity and Cauchy stress tensor, x^α is the spatial coordinate and $\frac{d}{dt}$ is the time derivative taken in the moving Lagrangian frame. In Eqs. (1)–(4), the effect of heat conduction is neglected assuming that the deformation process is locally adiabatic.

2.2. Constitutive model

The stress component in Eqs. (2) and (3) may be written in terms of hydrostatic and deviatoric stresses as Libesky and Petschek [14],

$$\sigma^{\alpha\beta} = P\delta^{\alpha\beta} - S^{\alpha\beta}, \tag{5}$$

where P and $S^{\alpha\beta}$ are respectively the pressure and the components of the traceless symmetric deviatoric stress tensor. The equations involved in calculating P and $S^{\alpha\beta}$ are given in the following subsections.

2.2.1. Pressure

The pressure in Eq. (5) may be calculated through an equation of state (EOS), which is generally a functional form of two or more thermodynamical properties (such as temperature, pressure, volume or internal energy) associated with the physics of the problem. For solids there is no general EOS that is appropriate for all materials and circumstances. The Mie–Gruneisen EOS is widely used in almost all hydrocodes [33]. For a comprehensive description of EOS for solids and other materials one may refer to Elizer et al. [4]. In this paper two EOS are used.

For elastic problem, pressure is assumed to vary linearly with compression ratio as,

$$P(\rho) = K\left(\frac{\rho}{\rho_0} - 1\right), \tag{6}$$

where, K is the bulk modulus and ρ_0 is the initial mass density.

For the elastic–plastic problem, the Mie–Gruneisen EOS [15], described below, is used.

$$P(\rho, e) = \left(1 - \frac{1}{2}\Gamma\eta\right)P_H + \Gamma\rho e, \tag{7}$$

where

$$P_H = a_0\eta + b_0\eta^2 + c_0\eta^3 \quad \text{for } \eta > 0 \quad \text{and} \quad P_H = a_0\eta^3, \quad \eta < 0, \tag{8}$$

$$\eta = \left(\frac{\rho}{\rho_0} - 1\right), \tag{9}$$

$$a_0 = \rho_0 C^2, \tag{10}$$

$$b_0 = a_0[1 + 2(S-1)], \tag{11}$$

$$c_0 = a_0[2(S-1) + 3(S-1)^2]. \tag{12}$$

Here, S and C, respectively, denote the linear shock-velocity and the particle-velocity parameters to describe the Hugoniot fit and Γ is the Gruneisen parameter.

2.2.2. Deviatoric stress

The deviatoric part of the Cauchy stress rate tensor may be written as,

$$\dot{S}^{\alpha\beta} = \mu\left(\dot{\varepsilon}^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\varepsilon}^{\gamma\gamma}\right), \tag{13}$$