2017 International Conference on Cyber-Enabled Distributed Computing and Knowledge Discovery

SIRS Model and Stability based on Open Cyber Ecosystem

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Abstract—**For an open cyber ecosystem, system elements will face the complexity and uncertainty to satisfy collective security defense. In the paper, a novel susceptible-infected-removed-susceptible (SIRS) model is proposed based on an open cyber ecosystem. More specifically, the factors of node increasing and decreasing are introduced to the traditional SIRS model, and system dynamics equations are derived. Via a Routh-Hurwitz stability criterion, the stability conditions and requirements are further analyzed. Theoretical analysis and simulations results show that SIRS epidemic spreading in an open cyber ecosystem can be well controlled by modulating the number of nodes in case of node increasing and decreasing.**

Keywords-cyber ecosystem; SIRS; model; stability

I. INTRODUCTION

A cyber ecosystem comprises a variety of diverse participants, the government, private firms, institutions, individuals, and cyber devices that interact for multiple purposes [1]. In reality, such a cyber ecosystem is a complex but open system. In the cyber ecosystem, nodes increasing and decreasing dynamically, the requirements of the cyber reconnaissance, attack, and defense will be satisfied. Then the process will unavoidably affect traditional SIRS model, and cyber ecosystem collective security defense. In previous research, achievements are mostly concentrated on close cyber environment, for example, dynamics of n impulsive delay and variable coefficients in a susceptible-infected (SI) model [2], the stability and bifurcation of epidemic spreading in a susceptible-infected-susceptible (SIS) model [3], the maximum of infective ability and minimum of network degree on susceptible-infected-removed (SIR) model [4], the dynamic evolution and influence factors in a susceptible-infected-removed-susceptible (SIRS) model [5-6]. However, a cyber ecosystem is essentially a dynamical and open complex system, and the number of cyber nodes increasing and decreasing is not constant [7]. With the development of cyber ecosystem, there are series of variety about characteristics of locally and globally in cyber ecosystem.

In the paper, a novel SIRS model is presented based on an open cyber ecosystem. In part Ⅱ, the SIRS model is established, and system dynamics equations are proposed. In part Ⅲ, the stability of the system is studied. Finally, simulations are given in part Ⅳ.

II. SIRS MODEL

In an open cyber ecosystem, cyber nodes increasing and decreasing are the reflection and representation. Some cyber nodes are eliminated naturally, or dilapidated by the factors of environment, human, and epidemic attack [1, 7], and thus new cyber nodes with special functions may be employed/increased to maintain the capability and dynamical balance of cyber ecosystem. In some cases, cyber nodes that have special function and assignments will be increased and decreased to satisfy new missions and designed capabilities in the process of cyber operation [8].

Based on traditional SIRS model and the mechanism of open cyber ecosystem, a novel model is presented and shown in Fig.1.

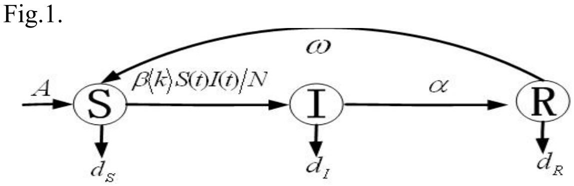

Figure 1. Novel SIRS model based on open network system

In Fig.1, A is denoted the number of cyber node increasing, and d_S, d_I, d_R are respectively the elimination rates of susceptible nodes, infected nodes, and removed nodes. The parameter β, is the rate of nodes contact with cyber epidemic, α is the rate of the capability of antiepidemic, ω is the rate of the capability of antiepidemic is decreased, and $\langle k \rangle$ is the cyber degree.

Therefore the corresponding system dynamics equations for the model can be given as:

$$\begin{aligned} \frac{dS(t)}{dt} &= A - \beta\langle k \rangle S(t)I(t)/N - d_S S(t) - \gamma S(t) + \omega R(t) \\ \frac{dI(t)}{dt} &= \beta\langle k \rangle S(t)I(t)/N - \alpha I(t) - d_I I(t) \\ \frac{dR(t)}{dt} &= \alpha I(t) + \gamma S(t) - \omega R(t) - d_R R(t) \end{aligned} \tag{1}$$

In Eq. (1), at the time t, $S(t)$ is the number of susceptible nodes, $I(t)$ is the number of infected nodes, $R(t)$ is the number of removed nodes, N is the total number of network nodes, $\beta\langle k \rangle S(t)I(t)/N$ is the rate of susceptible nodes that suffered from epidemics infected [5].

Assumed that the environment is the same with its influence in local area cyber ecosystem and randomly mutilation, elimination rates of nodes in different state are the same, i.e. $d_S = d_I = d_R = d$.

978-1-5386-2209-4/17 $31.00 © 2017 IEEE
DOI 10.1109/CyberC.2017.12

III. Stability Analysis

To solve the balance point about Eq. (1), let $\frac{dS(t)}{dt}=0$, $\frac{dI(t)}{dt}=0$, $\frac{dR(t)}{dt}=0$. One balance point of Eq. (1) can be obtained as:

$$P^0(S^0,I^0,R^0)=\left(\frac{A}{d},0,0\right) \tag{2}$$

And another balance point is

$$P^1(S^1,I^1,R^1)=\left(\frac{N(d+\alpha)}{\beta\langle k\rangle},I^1,\frac{\alpha}{(\omega+d)}I^1\right) \tag{3}$$

In Eq. (3), $I^1=\frac{(d+\omega)[\beta\langle k\rangle A-d(\alpha+d)N]}{\beta\langle k\rangle[(d+\alpha)(d+\omega)-\alpha\omega]}$, and the basic reproduction number of epidemics $R_0=\frac{d(\alpha+d)N}{\beta\langle k\rangle A}$, results can be gain as $I^1=\frac{A(d+\omega)[1-R_0]}{[(d+\alpha)(d+\omega)-\alpha\omega]}$, that if and only if $R_0\ge 1$, Eq. (1) only has disease-free equilibrium point P^0, and if and only if $R_0<1$, Eq. (1) only has endemic equilibrium point P^1. Jacobian matrix of the random equilibrium point P^* can be calculated:

$$J\left(P^*\right)=\begin{bmatrix} \frac{-\beta\langle k\rangle}{N}I-d & -\frac{\langle k\rangle S\beta}{N} & \omega \\ \frac{\beta\langle k\rangle}{N}I & \frac{\langle k\rangle S\beta}{N}-d-\alpha & 0 \\ 0 & \alpha & -\omega-d \end{bmatrix} \tag{4}$$

Theorem 1 If $R_0\ge 1$, disease-free equilibrium point P^0 is locally asymptotically stable, and if $R_0<1$, disease-free equilibrium point P^0 is unstable.

Proof Due to Eq. (4), Jacobian matrix of disease-free equilibrium point P^0 can be calculated:

$$J\left(P^0\right)=\begin{bmatrix} -d & -\frac{A\beta\langle k\rangle}{Nd} & \omega \\ 0 & \frac{A\beta\langle k\rangle}{Nd}-d-\alpha & 0 \\ 0 & \alpha & -\omega-d \end{bmatrix} \tag{5}$$

Eigenvalue determinant of $J(P^0)$ is:

$$\left|\lambda I-J\left(P^0\right)\right|=\begin{vmatrix} \lambda+d & \frac{A\beta\langle k\rangle}{Nd} & -\omega \\ 0 & \lambda-\frac{\beta\langle k\rangle A}{Nd}+d+\alpha & 0 \\ 0 & -\alpha & \lambda+\omega+d \end{vmatrix} \tag{6}$$

Let $\left|\lambda I-J(P^0)\right|=0$, eigenvalue polynomial of $J(P^0)$ is:

$$(\lambda+d)(\lambda+d+\omega)\left[\lambda-\frac{\beta\langle k\rangle A}{Nd}+d+\alpha\right]=0 \tag{7}$$

Three latent roots of Eq. (7) are $\lambda_1=-d$, $\lambda_2=-(d+\omega)$, and $\lambda_3=\left[\beta\langle k\rangle A-Nd(d+\alpha)\right]/Nd$. If $R_0\ge 1$, three eigenvalues of Eq. (7) are less than zero, and then disease-free equilibrium point P^0 is locally asymptotically stable. And if $R_0<1$, one of eigenvalues $\lambda_3>0$, there is one eigenvalue of Eq. (7) is more than zero, so disease-free equilibrium point P^0 is unstable.

As shown in Proof 1, in cyber operation, when infection capability of cyber epidemic doesn't reach the cyber security guard threshold, cyber security is an advantage, epidemic always dies out, and there are no inflected nodes.

Theorem 2 If $R_0<1$, the endemic equilibrium point P^1 is locally asymptotically stable.

Proof Jacobian matrix of endemic equilibrium point P^1:

$$J\left(P^1\right)=\begin{bmatrix} -\frac{\beta\langle k\rangle}{N}I^1-d & -d-\alpha & \omega \\ \frac{\beta\langle k\rangle}{N}I^1 & 0 & 0 \\ 0 & \alpha & -\omega-d \end{bmatrix} \tag{8}$$

Eigenvalue determinant of $J(P^1)$ is:

$$\left|\lambda I-J(P^1)\right|=\begin{vmatrix} \lambda+\frac{\beta\langle k\rangle}{N}I^1+d & d+\alpha & -\omega \\ -\frac{\beta\langle k\rangle}{N}I^1 & \lambda & 0 \\ 0 & -\alpha & \lambda+\omega+d \end{vmatrix} \tag{9}$$

When $\left|\lambda I-J(P^1)\right|=0$, eigenvalue polynomial of $J(P^1)$ is: