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Research Paper

# An algorithm for solving steady-state heat conduction in arbitrarily complex composite planar walls with temperature-dependent thermal conductivities

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## HIGHLIGHTS

- An algorithm for solving 1D heat conduction in arbitrary planar walls is provided.
- The algorithm is general and describes the wall topology as an algebraic equation.
- The problem is solved by overloading the algebraic operators in the equation.
- Temperature-dependent thermal conductivity of the materials is accounted for.
- The model handles conductive, convective, radiative, and contact thermal resistances.

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## ABSTRACT

An algorithm for solving steady-state heat conduction problems in arbitrarily complex composite walls is presented. *Per se*, steady-state heat conduction across a wall can easily be solved by hand. Yet, in practical applications the wall structure is often complex enough to deter such an approach if a finer yet simple analysis of the thermal bridges is of interest. Moreover, if high-temperature applications are involved, the additional complexity of including time-dependent thermal conductivity must be considered. Thus, a general methodology for solving arbitrary topology walls, involving any kind of thermal resistances in series and in parallel is discussed. While such a problem is formally simple to solve for a given wall following the theory, its algorithmic generalization is not. A method is provided, involving a program written in python language. The focus of the work is mainly on the algorithmic point of view: a simple way for the assessment of the wall topology and for the resolution of the heat conduction problem originating is sought. Temperature-dependent thermal conductivity of the materials is addressed, resulting in the need of evaluating the heat fluxes and the average temperature at each thermal resistance.



## 1. Introduction

The analysis of heat conduction across walls is of importance in many practical applications, ranging from energy efficiency in buildings, to many industrial processes where hot fluids are involved. In such applications, the goal can either be to increase or to limit the heat transfer rate across a wall as, for instance, in heat exchangers or in industrial kilns respectively. Considering the latter, a proper thermal insulation is of great importance in high-temperature industrial processes, where the thermal powers at play are of the orders of several MW, and the thermal dissipations remarkable. Yet, the industrial needs often impose a very complex structure to the insulating walls. This may include several thermal bridges, whose global effect on the thermal dissipation is not always properly assessed in industry. *E.g.*, continuous industrial roller kilns for firing ceramic tiles can work up to 1200 °C. Their walls are made of different types of refractory bricks and other insulating materials. Yet, the wall surface includes room for, to cite a few, rollers, fibres for reducing rollers friction, burners, inspection windows, and other service compartments.

While a 2D or 3D analysis of the performance of such a complex wall would be welcome, its times and costs are often incompatible with the industrial needs. A simpler instrument is given by the one-dimensional approach. Indeed, the theory of 1D steady-state conduction is already well established and clearly exposed in any thermodynamic student's book [1,2]. Yet, the complexity of the actual walls found in many industrial applications, would benefit

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from a fast, simple, and general tool, able to solve any kind of wall topology from the thermal point of view. Such a tool would speed up the design of proper wall structures, and could also be coupled to optimization algorithms in order to find the optimal layout in terms of thermal insulation, resistance of the materials at high-temperature, costs, and other industrial needs.

Similar tools are used for the thermal analysis of buildings, yet in that case the walls are modelled with simple lumped thermal resistances in series at best. This issue is also raised in [3] where the 1D parallel path description of the building envelope is said to be prone to serious errors in the overall thermal performance evaluation due to the simplistic model adopted. In this field of investigation other issues, such as the thermal radiation and the transient thermal response, generally attract more attention.

This reflects also in the literature, where much attention has been devoted in recent years to transient heat conduction. In [4] the transient response of a multi-layered slab to sudden temperature variation is investigated. In [5] a lumped model for transient 1D heat conduction is proposed under the assumption of constant thermal properties, hypothesis that falls in the later work [6] even though a simple slab geometry is addressed. Similar issues are addressed in [7] with a modal-based approach, in [8] where the explicit Green's method is proposed and experimentally validated for solving multiple layers slabs, and in [9] where the fast convergence of a Laplace trasform-based approach is discussed.

Another common field of investigation in recent heat conduction research is given by the resolution of the inverse problem aiming at finding the temperature-dependent thermal conductivity of a material from temperature measurements at some point in the wall, for given initial and boundary conditions. The problem is usually treated as 1D, and the wall made of an homogeneous slab. Several methods have been proposed for such a problem, which is generally solved iteratively due to its nonlinear nature. In [10] a linearization procedure is suggested to avoid iterative solutions, while in [11] a functional approach is proposed. In [12] the solution is found by means of optimization through genetic algorithms, also allowing the thermal conductivity to be any kind of function of the temperature, while in [13] a conjugate gradient optimization method is proposed, properly modified for better accounting for the sensitivities. Finally, [14] focuses on inverse heat conduction problems in the nano-scale, modelled with the Boltzmann transport equation.

This paper presents a general method for automating the resolution of the heat conduction problem in arbitrarily complex planar walls, such as those used for insulation in high-temperature industrial processes (*e.g.* continuous roller kilns). Due to the continuous nature of the industrial process, transient effects are not of interest and are neglected. On the contrary, due to the high-temperatures involved, the effect of the temperature-dependent thermal conductivity of the materials is addressed. The solution is based on the classical theory of one-dimensional steady-state conduction, which is simple, well known, and established. Yet, to the author's knowledge, following this theory no method for the automatic solution of a generic wall, whatever its topology, has been given in the literature. The focus of the paper is thus not on the solution method, which is 1D on purpose, but on its generalization in the form of algorithm. To this aim, a python program has been written and tested and its algorithmic infrastructure will be discussed in the following. Examples are given, and the results found are accompanied by CFD analyses performed with the OpenFOAM package in order to test the pros and the cons of the 1D approach.

## 2. Problem statement

Let us consider an arbitrary composite planar wall, made of a number of layers put in series and in parallel between them. Let also the boundary condition be given by uniform temperature imposed at the walls surfaces, $T_{w,1}$ and $T_{w,2}$. For simplicity of explanation, no convective or radiative effects are addressed at this stage. Yet, the algorithm generalization including these contributions in terms of additional thermal resistances, in parallel with respect to each other, at the wall extremities is absolutely straightforward. For instance, let assume that $T_{w,1}$ is the temperature inside the industrial kiln, while $T_{w,2}$ is the ambient temperature, so that $T_{w,1} \gg T_{w,2}$.

First of all, it is necessary to define the structure of the wall in terms of material properties, and layers geometry (*i.e.* area and thickness). The layers connectivity issue will be addressed later on. So, we consider as input of the problem:

- a list of materials together with information on their thermal conductivity in function of the temperature, $\lambda(T)$,
- the list of the walls layers together with their areas $A$, thicknesses $L$, and materials they are made of.

This information, is enough for computing the thermal resistance of each layer $R_i$, once its average temperature $T_i$ is defined, from the classical steady one-dimensional heat conduction theory:

$$R_i = \frac{L_i}{\lambda_i(T_i)A_i}, \quad \forall\, i \in \text{layers}. \tag{1}$$

From this, recalling the analogy with the Ohm's law we have that, for each layer/thermal resistance:

$$\dot{Q}_i = \frac{\Delta T_i}{R_i}, \tag{2}$$

where $\dot{Q}_i$ is the thermal power flowing through the resistance, and $\Delta T_i$ the temperature jump between its extremities.

In particular, in the python code written by the authors the thermal conductivity is given by two vectors: one for the temperatures, and one for the thermal conductivities at the same temperatures. This comes from the fact that materials technical sheets usually report thermal conductivity information in this pointwise manner. The thermal conductivity at any temperature can then be interpolated: cubic spline interpolation is adopted in the present work. The fact is that the $T_i\, \forall i$ is not known *a priori*, and an iterative procedure is needed for adapting the thermal conductivities at the actual operating temperature of the layers.

At this point, a simple and general method for defining the connectivity of the layers must be defined. The idea is to define the connectivity in the form of an equation where the thermal resistances in series are represented with a $+$ sign, and those in parallel with a $\times$ sign. This approach presents a few advantages: first of all it is easy to define the wall connectivity, and also it can represent any kind of arbitrarily complex composite wall. Yet, the only practical way to solve the heat conduction problem is by parsing, with the due caution, the string defining the equation.

As an example, the wall and the equivalent thermal circuit shown in Fig. 1 is represented by the formula:

$$(R_1 + R_2 + R_3) \times (R_4 + R_5 \times R_6) + R_7. \tag{3}$$

### 2.1. Input consistency

The input strategy outlined above may lead to non-consistent input data, in the sense that parallels having different thickness and series having different areas are formally allowed. Even though this does not represent an issue for the algorithm operation, a consistency check can be implemented.

To do this, it is necessary to modify Eq. (3) so that its evaluation would return the overall wall thickness (or area) if thickness (or