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Research Paper

Heat transfer in bare and insulated electrical wires with linear temperature-dependent resistivity

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ABSTRACT

This article presents the evaluation of the electric current-carrying capability (ampacity) of an electrical wire, covered or not with an insulating material, carrying a direct electrical current, taking into account the electrical resistivity temperature-dependence of the wire core. The evaluation was based on a heat transfer model consisting of an infinitely long two-layer cylinder with temperature-dependent volumetric heat generation under boundary conditions of the first and third kinds.

A comparison between the closed form solutions reported in this manuscript versus the classical solutions, i.e. the ones in which the electrical resistivity is considered temperature-independent, showed that the they allowed for predicting the situations in which one must consider the temperature-dependent resistivity of the wire core on the ampacity and the influence of the convection coefficient of heat transfer. The closed form solutions were successfully validated using the ANSYS/Steady-state thermal software.

1. Introduction

The knowledge of the temperature distribution inside a cylinder or a composite cylinder is required in many areas of science and engineering [1–3]. These areas include the conduction of heat in nuclear fuel rods, external heating of rods and tubes in metallurgical processes, measurement of non-electrical quantities such as thermal conductivity of fluids (transient hot-wire method) and wind speed (hot-wire anemometers), internal heating of electrical wires (ampacity evaluation), etc.

Despite the growing use of sophisticated computational fluid dynamics software tools used by engineers for design and analysis purposes, the development of new closed form analytical solutions in heat transfer continue to be important [4,5]. An exact analytical solution of the problem of heat transfer in pin-fins of infinite length made of a high thermal conductivity core with a low thermal conductivity coating heated through the base is reported in [4]. The authors proposed as simplifying assumptions no internal heat generation and materials with constant thermophysical properties. An exact analytical solution of the problem on the 3D transient temperature distribution in a cylinder with multiple radial layers with a time-dependent, spatially non-uniform internal volume heat source is documented in [5]. The transient temperature distribution was calculated with the use of the eigenfunction expansion method. First and second kind boundary conditions in the angular and axial directions of the cylinder and for the nonhomogeneous third kind boundary conditions in the radial direction were considered.

Exact analytical heat transfer solutions can provide insights for designing an equipment, simplify the verification process of numerically based solutions of heat transfer problems, optimize experimental heat transfer investigations planning, etc.

This manuscript presents closed form analytical solutions for one-dimensional steady-state conduction of heat in infinitely long two-layer cylinder with temperature-dependent volumetric heat generation under two boundary conditions on the free surface: specified temperature (boundary condition of the first kind), and specified convection and radiation heat transfer (boundary condition of the third kind). The solutions were applied to determine the electric current-capability of two types of wires (a bare and an insulated) with temperature-dependent electrical resistivity carrying a direct electrical current. This is a challenging subject for systems protection in the industry. There are three major economic sectors where the results of this study can be applied: electric power transmission and distribution, power supply cable harness systems in the automotive industry, and electronic/electric equipment designs.

In electric/electronic assemblies, electrical power has to be delivered through a metallic wire to a load. In this process, the metallic wire heats up. If the electrical current associated exceed

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Nomenclature

a	coefficients related to the heat rate per unit volume, $W\,m^{-3}$
A	surface area transverse to current flow, m^2
b	coefficients related to the heat rate per unit volume, $W\,m^{-3}\,K^{-1}$
c	specific heat, $J\,kg^{-1}\,K^{-1}$
D	wire diameter, m
d	wire core diameter, m
h	heat transfer coefficient, $W/m^2\,K$
I	electrical current intensity, A
J	electrical current density, $A\,m^{-2}$
J_0, J_1	Bessel's functions of the first kind, –
k	thermal conductivity, $Wm^{-1}\,K^{-1}$
L	length of the wire, m
N_0	Neumann's function of the first kind and zero order, –
Nu	Nusselt's number, –
Pr	Prandtl's number, –
q'	heat rate per unit length, $W\,m^{-1}$
q''	heat rate per unit area, $W\,m^{-2}$
q'''	heat rate per unit volume, $W\,m^{-3}$
r	radius, m; radial coordinate, m
R	electrical resistance, Ω
Ra	Rayleigh's number, –
T	temperature, K
t	time, s
$\mathbf{x}$	position, m

Greek symbols

α	absorptivity of the wire core or wire cover, –
δ	thickness of the insulation cover, m
γ	coefficient of a polynomial interpolation first-order, $\Omega\,m$
ε	emissivity of the wire core or wire cover, –
η	coefficient of a polynomial interpolation first-order, $\Omega\,m\,K^{-1}$
ρ_D	density, $kg\,m^{-3}$
ρ_0	electrical resistivity at 293 K, $\Omega\,m$
ρ	electrical resistivity, $\Omega\,m$
σ	Stefan-Boltzmann constant ($=5.67 \times 10^{-8}$), $W\,m^{-2}\,K^{-4}$

Subscripts

1	refers to electrical conductor core of the wire
2	refers to insulation cover of the wire
c	refers to heat transfer by convection
r	refers to heat transfer by radiation
s	refers to free surface of the wire core or cover
∞	refers to region outside the boundary layer

Abbreviations

BC	boundary condition
PVC	polyvinylchloride

too much a given value, the wire core and its surrounding (for instance, its insulation cover) might fail. This is the reason why it is relevant to know the temperature inside the wire core and surrounding to a given electrical current as accurate as possible.

For many years, increasing attention has been devoted to the analysis of heat transfer in electrical conductors used by public and private utilities to power transmission and distribution [6–16]. In these cases, the conductors form long lines and consequently a huge amount of heat is generated.

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In greater metropoles, where there is a large concentration of residential and commercial buildings, electricity is usually delivered to the city by overhead cables and through buried cables. The electric current carrying capability of overhead cables [6–9] is normally affected by ambient air-temperature, wind direction and speed, conductor size, intensity of solar radiation, and physical properties of the cable. On the other hand, ampacity of buried cables [7,10–16] is affected by conductor size, soil thermal resistivity, bonding arrangement, depth of the backfilling, underground temperature distribution, and ambient air-temperature.

Recently, due to the advance of the embedded electronic systems in vehicles, more and more electric power is required; for either safety reasons (e.g., power door locks) and/or comfort reasons (e.g., power windows). Therefore, the optimization of the system responsible for the power supply of the cable harness becomes a very important issue. The large amount of cables having a few meters of length confined in small spaces in cars has motivated a series of studies about the ideal diameter, material, and size of these cables, and consequently their weight. The thermal analysis normally considers various parameters, such as ambient air-temperature, thickness, and material of the insulation cover when considering the maximum current intensity that can be carried and the heat to be dissipated [17,18].

Electronic equipment has a set of components such as inductors and transformers. These devices heat up and the heat generated must be removed. Also important is the optimization of the amount of heat generated in the interconnections of microelectronic components [19]. In conclusion, a comprehensive literature survey, represented in this manuscript by [6–18], revealed that closed form analytical solutions calculation considering temperature-dependent electrical resistivity is not used in ampacity calculation despite the error that can be committed.

The manuscript is organized as follows. Section 2 presents the mathematical formulation of the physical problem, Section 3 the exact analytical solutions of the physical problem, Section 4 the numerical results, and Section 5 the main conclusions of the investigation.

2. Mathematical formulation of the physical problem

2.1. Geometry of the wire

Fig. 1 shows a sketch of the wire. In this work, it was considered that both the wire core and the insulation cover are made of isotropic and homogeneous materials. Heat is generated in the wire core and dissipated by convection and radiation from the free surface.

2.2. Governing equations

The equation for the conduction of heat in an isotropic medium is the following [20]:

$$\nabla \cdot [k(T)\nabla T] + q'''(\mathbf{x}, T, t) = \rho_D c(T)\frac{\partial T}{\partial t}, \tag{1}$$

where k is the thermal conductivity, ρ_D is the density, c is the specific heat, T is the temperature, $\mathbf{x}$ is position, t is time, and q''' is the energy generation rate per unit of volume.