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Research Paper

Analytical solution to steady-state temperature field for straight-row-piped freezing based on superposition of thermal potential

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HIGHLIGHTS

- Thermal potential superposition - viable in solving steady-state temperature field.
- Analytical solution to temperature field by triple-row-piped freezing is derived.
- The analytical solution compares very well with the computational results.
- Frozen soil wall thickness can be determined by measuring one point's temperature.

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ABSTRACT

The existing analytical solutions to steady-state temperature field for straight row-piped freezing in artificial ground freezing cover single-piped, multi-piped, single-row-piped and double-row-piped freezing, yet they are not sufficient to meet the needs of construction projects because triple-row-piped or even multi-row-piped freezing are increasingly adopted. In this paper, the original engineering problems of the temperature field of single-piped and row-piped freezing are firstly converted to the solving of a plain Laplace equation under specified boundary conditions. Based on superposition of thermal potential, the analytical solutions to steady-state temperature field for single-piped, single-row-piped, double-row-piped freezing are deduced in succession and then the solution for triple-row-piped freezing is obtained, which is then validated through comparison with the thermal numerical analysis. Furthermore, the formula for calculating the thickness of triple-row-piped frozen soil wall is proposed, which is important in construction management of artificial ground freezing.

1. Introduction

Artificial ground freezing (AGF) is a technique that converts the soil water into ice with artificial refrigeration technology, creating a strong, watertight frozen soil wall which can be used as temporary work. As a method of ground improvement, AGF has been widely adopted in tunnel excavation, mine shaft sinking and municipal engineering. However, some problems remain to be solved in obtaining analytical solutions of temperature field in AGF, such as temperature field in triple-row-piped or even multiple-row-piped freezing. The calculation of temperature field is the pre-requisite in studying the temperature distribution, i.e. it is the significant basis for the design, construction and safety management in AGF.

Since the middle of last century, several analytical solutions to steady-state temperature field in AGF had been derived gradually. Trupak [1], studied the temperature field formed by single freezing pipe and obtained the analytical steady-state solution to single-piped frozen temperature field. What's more, he provided the analytical steady-state solution to single-row-piped frozen temperature field according to the geometrical relationship between two adjacent frozen soil columns. The results, however, were much different from experimental data because of his ignorance of interaction between two adjacent pipes. Bakholdin [2] considered that once two adjacent frozen soil columns were merging, the wave-shaped boundary would soon turn into a flat one. Based on the theory of analogy between thermal and hydraulic problems, he obtained the analytical solutions to single-row-piped and double-row-piped steady-state frozen temperature field, which were proved to be exact enough. Sanger and Sayles [3] presented a simplified formula for single-row-piped frozen temperature field, but this formula bears certain inaccuracy [4]. Tobe and Akimoto [5]

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Nomenclature

Notation

x, y	coordinate
q_x	density of the heat flux in unit time in the direction of x
q_y	density of the heat flux in unit time in the direction of y
q	density of heat flux of a point at the circumference of a circle of radius r around a point cold source
q_{ci}	heat flux of the i-th freezing pipe
Φ_e	thermal potential of any point on the boundary of the frozen soil wall
Φ_f	thermal potential of a point at the outer surface of the freezing pipe
$\Phi(x,y)$	thermal potential of an arbitrary point (x,y)
k	thermal conductivity of the ground
M	a point at the circumference of the cold source
E	nearest point to M at the boundary of the frozen soil wall
r	radius of a circle surrounding the cold source
r_i	distance between M and the center of the freezing pipe
ξ_i	distance between the i-th freezing pipe and point E
ξ	thickness of the frozen soil wall
C	integral constant
A, B_1, B_2, B_3	intermittent variable
D_1, D_2, D_3	intermittent variable
r_0	radius of the freezing pipe
T	distribution of temperature field
T_f	surface temperature of the freezing pipe
T_0	temperature at the boundaries of the frozen soil, i.e. the freezing temperature of soil
T_s	initial temperature of soil
L	row spacing
l	distance between two adjacent freezing pipes
η	ratio of the heat flux of the first-row pipes to that of the second-row pipes
ω	parameter used for distinguishing between the aligned row-piped freezing and the staggered row-piped freezing
f	function name
W	thickness of the triple-row-piped frozen soil wall

derived an analytical solution to multi-piped frozen temperature field.

Until now, there have been no precise analytical solutions for triple-row-piped or multi-row-piped frozen temperature field. In this paper, an analytical solution to triple-row-piped frozen temperature is obtained using the superposition of thermal potential. When the frozen soil wall develops to a certain thickness (greater than half of the spacing between freezing pipes), the rate of heat conduction becomes very slow, and thus the influence of time becomes negligible in the investigation of the characteristics of the temperature field of the frozen soil wall. Because of this, the steady-state temperature field is adopted to study the temperature field of the frozen soil wall.

2. Mathematical model

The original three-dimensional heat conduction problem can be simplified as a two-dimensional one in a real AGF project in that the longitudinal length of a freezing pipe is by far greater than its diameter and its surface temperature actually changes very slowly in the longitudinal direction. Then, by the first law of thermodynamics, the equation of two-dimensional steady-state heat conduction is:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 0 \tag{1}$$

where q_x, q_y are the densities of the heat flux in unit time in the directions of x and y, respectively.

We introduce a parameter Φ called thermal potential here, which is given by:

$$\Phi = kT \tag{2}$$

where k is the coefficient of thermal conductivity of soil and T is the distribution of temperature field.

Using Fourier's law, we get:

$$q = -k\frac{\partial T}{\partial x} \tag{3}$$

Substituting Eq. (2) into Eq. (3) yields:

$$q = -\frac{\partial \Phi}{\partial x} \tag{4}$$

We assume that the soil is isotropic in thermal physics, so k is identical in all directions. And k is the same once the temperature drops below the freezing point (this assumption approximates to the real case). Then, substituting Eq. (4) into Eq. (1) yields:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0 \tag{5}$$

The polar form of it can be written as:

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\Phi}{\mathrm{d}r}\right) = 0 \tag{6}$$

Now the original engineering problem has been converted to the solving of a plain Laplace equation under specified boundary.

3. The principle of potential superposition

The principle of potential superposition is: if there are a number of (or numerous) points of potential source in a plane, the potential of each point is equal to the sum of potentials generated at the very point by all the point potential sources. In electromagnetics, the principle of potential superposition is known as the superposition of electric potential [6] and in hydraulics, the superposition of "velocity potential" or water head [7]. This method has been widely used to solve the problem of Laplace under certain boundary conditions in both fields [8–10]. However, even though it applies to solving similar problems in thermodynamics, as has been proved by related research [11–13], few examples of application can be found in this field.

In an AGF problem, a freezing pipe can be treated as a point cold source because the diameter of a freezing pipe is much smaller than the thickness of frozen soil wall or the spacing between freezing pipes. The absorbed heat of a point cold source in unit time, defined as q_c, will decrease the thermal potential of all points in the plain and this kind of reduction of thermal potential can be superposed when there are a number of such point cold sources. Therefore, the thermal potential of each point equals to the superposition of that generated at the point by all the point cold sources.

In accordance with the assumptions above, if we draw a circle of radius r around the point cold source and define the density of heat flux of a point at its circumference as q, then