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Elementary ship models and farfield waves

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ABSTRACT

Kelvin's classical 1-point ship model, the basic 2-point models of a monohull ship and a catamaran previously studied via elementary geometrical considerations, optimal versions of these two basic 2-point models, and a 4-point model of a catamaran (not considered previously) are analyzed within a common theoretical approach based on potential flow theory, the Green function associated with the Kelvin–Michell linear boundary condition at the free surface, the related Fourier representation of farfield ship waves, and the farfield stationary-phase approximation. The analysis shows that the apparent wake angle associated with the highest waves found inside the cusps of the Kelvin wake as a result of interferences among the divergent waves created by a ship at a high Froude number can be realistically predicted via elementary ship models. In particular, the apparent wake angles for monohull ships and catamarans can be predicted very well via a 2-point model or a 4-point model. Moreover, these models provide basic insight into main features of the farfield waves created by fast ships. Notably, differences between the amplitudes of the bow and stern waves are shown to have no influence on the occurrence of constructive or destructive interferences, although they affect the intensity of wave-interference effects (strong if the amplitudes of the bow and stern waves are commensurate, weak otherwise). Another notable conclusion is that lateral interference effects become more important as the Froude number increases. Indeed, lateral interferences between the twin hulls of a catamaran dominate longitudinal interferences between the fore and aft of the ship at Froude numbers greater than about 1. However, longitudinal interferences between the bow and the stern waves remain dominant for common fast monohull ships.

1. Introduction

The farfield waves due to a monohull ship or a catamaran of length L that travels at a constant speed V along a straight path, taken as the X axis, in calm water of large depth and horizontal extent are considered. The Froude number F is defined as

$$F = V/\sqrt{gL} \tag{1}$$

where g is the acceleration of gravity. The waves created by the ship are observed from a Cartesian system of coordinates (X, Y, Z) attached to the ship. The undisturbed free surface is taken as the plane $Z = 0$, and the Z axis is vertical and points upward. The X axis points toward the ship bow. The farfield ship waves are analyzed within the usual framework of linear potential flow theory, as in numerous studies since Kelvin's classical analysis [1]. This basic flow model is realistic, and the only available practical option, to analyze farfield ship waves.

Within this theoretical framework, the flow created by the ship can be represented in terms of a distribution of sources (and possibly dipoles) over the mean wetted surface of the ship hull, as is well known; e.g. [2]. Far behind the ship, this hull-surface distribution of sources can be modeled as a point source (or point pressure) located at the centroid of the ship waterplane. This 1-point wavemaker model of farfield ship waves, used by Kelvin in his analysis, yields important fundamental insight into dominant features of farfield ship waves. In particular, Kelvin's elementary 1-point ship model shows that farfield ship waves consist of two sets of waves, called transverse and divergent waves, that exist inside a wedge with angle $2\,\psi^K \approx 39^\circ$ aft of the ship.

The 1-point ship model evidently cannot account for interferences among the elementary waves (contained within the Green function G associated with the boundary condition at the free surface) that radiate from every point of the ship hull surface, in

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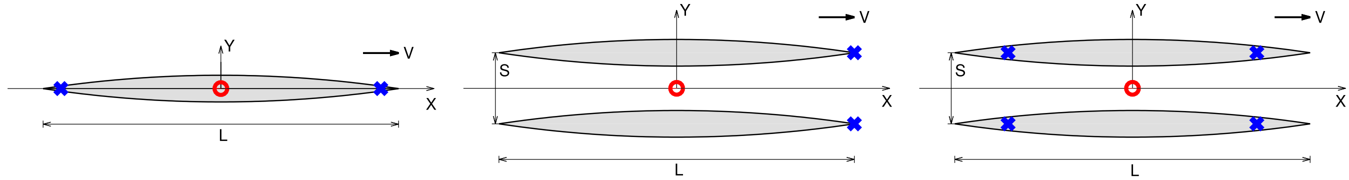

Fig. 1. The figure on the left depicts a top view of a monohull ship of length L that travels in calm water at a constant speed V along the x axis. This figure also shows the 1-point ship model (red bullet) used in Kelvin's classical analysis, and the 2-point ship model (two blue crosses) used to analyze interferences between the waves created by the bow and the stern of the ship. The figures in the center and on the right similarly depict top views of a catamaran with identical twin hulls of length L, separated by a distance S, and the 1-point ship model (red bullet). In addition, these two figures show the 2-point ship model (two blue crosses) used to analyze interferences between the waves created by the twin bows of the catamaran, and the more precise 4-point ship model (four blue crosses) that also accounts for longitudinal interferences between the waves created by the bows and the sterns of the catamaran.

accordance with the classical representation of potential flows in terms of surface distributions of sources given in e.g. [2]. Wave-interference effects can be approximately taken into account via the 2-point ship models considered in [3] for deep water, and in [4,5] for the more general (and considerably more complicated) case of uniform finite water depth. These 2-point models of farfield ship waves consist of a point source at the bow and a point sink at the stern of a monohull ship, or a pair of point sources (or sinks) at the twin bows (or sterns) of a catamaran with identical twin hulls, as is illustrated on the left and in the center of Fig. 1.

The 2-point models of a monohull ship and a catamaran considered in [3] are analyzed again here via a different, more formal, approach. Specifically, the analysis given in [3] is based on elementary geometrical interference relations that only rely on the dispersion relation (like Kelvin's classical analysis for the 1-point ship model). Indeed, the elementary analysis given in [3] does not involve the wave component W in the Green function G, and moreover does not consider the amplitudes of the waves created by the bow and the stern of a ship. The more precise analysis considered in the present study accounts for the amplitudes of the bow and stern waves, and involves the wave component W in G and the farfield stationary-phase approximation. This more elaborate analysis corroborates and refines the results of the elementary analysis previously considered in [3], and provides further insight. In particular, the analysis given here shows that differences between the amplitudes of the bow and stern waves have no influence on the occurrence of constructive or destructive interferences, although they affect the intensity of wave-interference effects (strong if the amplitudes of the bow and stern waves are commensurate, weak otherwise).

The extensive numerical studies, reported in [6–8], of the farfield waves created by monohull ships and catamarans, modeled by means of realistic hull-surface distributions of sources in accordance with Hogner's classical theory [9], show that the two elementary 2-point ship models depicted on the left and in the center of Fig. 1 yield realistic predictions of apparent wake angles associated with the highest waves that result from constructive interferences of divergent waves at high Froude numbers. The apparent wake angles predicted by these two basic 2-point ship models are also shown in [8,10] to be consistent with the observations of narrow ship wakes reported in the literature (a list of references to these observations of apparent narrow ship wakes may be found in [3] and the other previously-noted studies of wave-interference effects on farfield ship waves). However, the elementary 2-point model of a catamaran used in [3] evidently cannot account for the longitudinal interferences between the waves created by the bows and the sterns of the catamaran and is not accurate at low Froude numbers, for which longitudinal interference effects are important. In particular, [11] shows that the 2-point model of a catamaran poorly predicts the wavelength of the highest waves created by the ship at $F < 1$.

The analysis of the basic 2-point model of a catamaran depicted in the center of Fig. 1 and considered in [3] is then extended here. Specifically, a generalized 2-point model of a catamaran in which the distance between the pair of point sources is allowed to differ from the distance between the twin bows of the catamaran is considered. A generalized 2-point model of a monohull ship in which the distance between the point source and the point sink can significantly differ from the distance between the bow and the stern of the ship is considered as well. Optimal 2-point models that yield apparent wake angles, where the highest divergent waves are found, that agree with the apparent wake angles predicted in [6] and [7] via the Hogner source-distribution model are then considered. However, a realistic model of a catamaran requires two point sources near the bows and two point sinks near the sterns of the twin hulls of the catamaran, as is illustrated on the right of Fig. 1. This more realistic 4-point model of a catamaran, not considered previously, is analyzed in the present study.

Thus, elementary ship models – the basic 2-point models of a monohull ship and a catamaran previously considered in [3] via an elementary geometrical analysis, optimal versions of these two basic 2-point models, and the 4-point model of a catamaran – are analyzed within the classical framework of potential flow theory, the Green function associated with the linear Kelvin–Michell boundary condition at the free surface, the related Fourier representation of farfield ship waves, and the farfield stationary-phase approximation. The analysis shows that these elementary ship models can realistically predict the highest waves found inside the cusps of the Kelvin wake as a result of interferences among the divergent waves created at the bow and the stern of a fast ship.

The 1-point, 2-point and 4-point ship models that are considered here are the most basic and simplest models that can be used to analyze farfield ship waves. Other simple flow models can provide further insight and have been used. In particular, a ship is modeled via a distribution of pressure over the free surface in [12,13], or via a distribution of sources over the ship hull surface in [6,7]. This hull-surface source-distribution model, due to Hogner, arguably is the most realistic of the simple flow models that can be used to analyze farfield ship waves. References to numerous other studies of farfield ship waves can be found in [6,7,10,11].

2. Farfield waves due to a point source

The waves created by a ship are observed at flow-field points

$$\widetilde{\mathbf{x}} \equiv (\widetilde{x},\widetilde{y},\widetilde{z}) \equiv (\widetilde{X},\widetilde{Y},\widetilde{Z})\, g/V^2 \tag{2a}$$

The Cartesian coordinates $\widetilde{x}$ and $\widetilde{y}$ can be expressed in the polar form

$$\widetilde{x} = -h\cos\psi \quad \text{and} \quad \widetilde{y} = h\sin\psi \tag{2b}$$

where the nondimensional horizontal distance $h \equiv Hg/V^2$ and the ray angle ψ are defined as

$$h \equiv \sqrt{\widetilde{x}^2+\widetilde{y}^2} \quad \text{and} \quad \tan\psi \equiv \widetilde{y}/(-\widetilde{x}) \tag{2c}$$

The ray $\psi = 0$ corresponds to the track $\widetilde{x} < 0, \widetilde{y} = 0, \widetilde{z} = 0$ of the ship.