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Effect of dispersed phase fraction on the drag coefficient of a droplet or a bubble in an idealized two-phase flow

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ABSTRACT

In the present work, the drag force on a fluid sphere surrounded by a second is determined. This information is essential to characterize dispersed flows. Most of two-phase flow models are empirically obtained from experiments involving isolated bubbles or droplets. Their applicability to flows with a high number of bubbles or droplets is questionable. Moreover, correlations that take into account the effect of local dispersed phase fraction, the ratio of occupied volume by the dispersed phase, are sparse.

This paper explores the drag on a fluid sphere inside an idealized flow. All spheres are assumed identical having the same velocity and equidistributed in space. We propose a new relation for the drag coefficient of a sphere (bubbles or droplets) depending on Reynolds number *Re*, dispersed phase fraction ε, viscosity ratio μ^* and density ratio ρ^*. Analytical and numerical results are compared with previous studies including experimental measurements. The results lead to a proposal for a general relation of the drag coefficient for a sphere inside a cloud of spheres.

In the idealized bubbly/droplet flow considered, the slip ratio is very small, and the flow around a sphere can often be characterized by Stokes approximation. In addition, dispersed phase fraction, ε, has a strong effect on drag essentially through confinement. The wake and hydrodynamic interactions between spheres are comparatively small. The proposed relation can be used to elaborate a two-phase flow model for bubbly or annular flows. This work proposes an improvement on the closure relations for the drag coefficient (C_D).

The dependence of the drag coefficient (C_D) with void fraction is of utmost importance to evaluate the stability of two-phase flows. The drag coefficient is a source term of the averaged Navier–Stokes equation.

The main conclusion is that dispersed phase fraction extends the range for which Stokes flow represents accurately the flow around bubbles/droplets.

0. Introduction

In the nuclear, hydroelectric and chemical process industries, 50% of components and piping elements operate with two-phase flows [1]. In order to evaluate the forces induced by two-phase flow, it is desirable to identify the parameters that govern the flow. For two-phase flow, each phase interacts with the other phase through interfacial forces. In the present paper, we propose to investigate one of the interfacial forces: the drag of a sphere inside a two-phase flow.

This information is essential to characterize finely dispersed bubbly flows, where small spherical gas bubbles are present in a continuous liquid phase. It is also useful to model annular flow, for which spherical droplets are dispersed in a continuous gas phase.

Using either Euler–Euler or Euler–Lagrange models some authors use the widely accepted closure correlations proposed by Tomiyama et al. [2,3]. As underlined by Darmana et al. most of the closures are empirically obtained from experiments involving single bubbles or droplets [4]. Their applicability to systems with high dispersed phase fraction is questionable. Moreover, from available correlations in the literature, very few take into account the effect of the local dispersed phase fraction. Finally Zenit and Sangani conducted experiments with a suspension of spherical bubbles. They concluded that discrepancies found with the drag of an isolated bubble are not accounted for in the currently available theories [5].

This paper explores the drag on a fluid sphere inside an idealized homogeneous flow (identical spheres having same velocities). Dispersed and continuous quantities will be denoted with respective subscripts $_d$ and $_c$. A priori, the drag coefficient (C_D) depends on fluid viscosities (μ_c, μ_d), fluid densities (ρ_c, ρ_d), sphere size (radius: a), dispersed phase fraction (ε—ratio of dispersed

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Nomenclature

Variables

a:	Sphere radius (m)
B_i, C_i, D_i:	Integration constants
f:	Force (N)
g:	Gravitational acceleration (m/s^2)
l:	Inter-sphere distance (m)
P:	Fluid pressure (Pa)
U, u, v:	Velocity (m/s)
ρ:	Mass density (kg/m^3)
γ:	Surface tension (N/m)
μ:	Viscosity (Pa s)
δ:	Dirac function
r, θ, ϕ:	Spherical coordinate

Subscripts

$_c$:	Continuous phase
$_d$:	Dispersed phase
$_s$:	Sphere
$_{r,\theta,\phi}$:	Spherical coordinates

Dimensionless numbers

$Re = \frac{2\rho_c U_s a}{\mu_c}$:	Reynolds number
$Bo = \frac{4\Delta\rho g a^2}{\gamma}$:	Bond Number
$\rho^* = \frac{\rho_d}{\rho_c}$:	Density ratio
$\mu^* = \frac{\mu_d}{\mu_c}$:	Viscosity ratio
ε:	Dispersed phase fraction
C_D:	Drag coefficient

Superscripts

$\langle X\rangle, \bar{X}$:	Averaged variable
X^*:	Dimensionless number
X^i:	Related to the interface

Other symbols are defined in the text

phase volume to total volume) and sphere velocity (U_s). Assuming there are no other parameters involved such as the presence of impurities and according to the Buckingham-π theorem, the drag coefficient (C_D) is a function of four independent dimensionless numbers. We chose the viscosity and density ratios $\mu^* = \mu_d/\mu_c$, $\rho^* = \rho_d/\rho_c$, the Reynolds number $Re = 2\rho_c U_s a/\mu_c$ and dispersed phase fraction ε. For bubbly flow the dispersed phase fraction is the void fraction whereas for droplet clouds the dispersed phase fraction is one minus the void fraction.

The goal of the present work is to find a general relation $C_D = F(\mu^*, \rho^*, Re, \varepsilon)$. Our results will be compared with previous studies. Several authors have proposed relations for an infinite medium, meaning that $C_D = F(\mu^*, \rho^*, Re, \varepsilon = 0)$ [6–11]. Some authors have, however, considered the dispersed phase fraction effect on bubble velocity [12,13].

The paper is divided into four sections. At first, a literature review of available drag coefficient models is discussed. In Section 2, new analytical solutions for Stokes and Euler flows depending on void fraction are presented. In Section 3, a general model of drag coefficient is presented and compared to numerical experiments. In Section 4, the proposed model is compared to experimental results. The main results are summarized in the conclusion section.

1. Drag coefficient and shape of fluid sphere

This first section introduces basic definitions and presents available correlations from the literature.

1.1. Drag correlations

Many studies proposed drag correlations [7–9,11–13]. No correlation proposed dependence of drag on fluid density. The fluid density has however an effect on the shape of the fluid sphere. As a reasonable qualitative prediction, a small fluid sphere, characterized by a small Bond number value $Bo = 4\Delta\rho g a^2/\gamma \ll 1$, can be considered as spherical. All drag coefficient relations presented in Tables 1 and 2 are based on the following definition of the drag force:

$$\boldsymbol{f_D} = -\frac{1}{2} C_D \rho_c \pi a^2 U_s \boldsymbol{U_s}. \tag{1}$$

As the viscosity ratio increases the shear stress at the sphere surface increases. A large viscosity ratio $\mu^* \gg 1$ corresponds to an attachment condition on its surface: a solid sphere or most of droplet. Clift et al. proposed a drag coefficient correlation for a solid sphere for $Re < 3.7 \times 10^5$ (see Eq. (a) in Table 1) [7].

For low Reynolds numbers, the analytical solution using Stokes approximation for a solid sphere leads to $C_D = 24/Re$ and for a zero stress shear condition $C_D = 16/Re$. Taylor and Acrivos propose a correction taking into account convective terms, which become predominant far away from the bubble, and the effect of viscosity ratio (see Eq. (b) in Table 1). The first correction term of this equation (b) corresponds to the equation correction proposed by Oseen in 1910 when $\mu^* \to \infty$ [11]. This relation is a good approximation of the drag coefficient for a solid sphere at the limit when μ^* tends to infinity and for low Reynolds numbers.

A null viscosity ratio ($\mu^* = 0$) corresponds to the ideal zero shear stress condition. Most of the bubbles present this condition on its surface characterized by $\mu^* \ll 1$. Mei et al. propose a relation assuming the ideal zero shear stress condition for any Reynolds number (see Eq. (c) in Table 1). Assuming potential flow, $Re \ll 1$ and $\mu^* \ll 1$, leads to the same limits as Mei et al. correlation (c) $48/Re$ [9]. For low Reynolds numbers, the Mei et al. relation (c) has the same limit than Taylor and Acrivos relation (Eq. (b) in Table 1 with $\mu^* = 0$).

For non-spherical shape, assuming potential flow and small departure from sphericity, Moore obtains a correction of the drag coefficient (d) depending on the aspect ratio (χ).

Zuber and Hench have proposed a similar relation (e) in Table 2 which, however, does not take into account the viscosity ratio [12]. Ishii and Zuber have also proposed a relation (f) in Table 2 for a droplet or small bubble at low Reynolds numbers which includes dispersed phase fraction dependence [13].

Taylor and Acrivos, Zuber and Hench, Ishii and Zuber [8,12,13] are in agreement for $\mu^* \gg 1$ and $\varepsilon = 0$. Some small bubbles, probably due to impurity, behave as solid particles (equivalent to $\mu^* \gg 1$).

The terminal velocity is determined by the equilibrium between the buoyancy force applied on the fluid sphere ($\Delta\rho g a^3$) and its drag force:

$$\begin{aligned}\frac{1}{2} C_D \rho_c \pi a^2 U_s^2 &= \frac{4}{3}\pi \Delta\rho g a^3 \Rightarrow U_s = \frac{1}{3}\frac{16}{C_D Re}\frac{\Delta\rho g a^2}{\mu_c} \\ &= \frac{1}{3}\frac{16}{C_D Re} U_0 \end{aligned} \tag{2}$$

where $\Delta\rho = \rho_c - \rho_d$ represents the density difference and U_0 is a typical "sphere velocity" expressed as $U_0 = \Delta\rho g a^2/\mu_c$. For zero