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Statistical behaviour of vorticity and enstrophy transport in head-on quenching of turbulent premixed flames

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ABSTRACT

The transport of vorticity and enstrophy in the near-wall region for head-on quenching of turbulent combustion by an isothermal inert wall has been analysed using three-dimensional Direct Numerical Simulation (DNS) data of statistically planar turbulent premixed flames characterised by various global Lewis numbers *Le* (ranging from 0.8 to 1.2) and turbulence intensities. In all cases the vorticity magnitude shows its maximum value at the wall and the vorticity magnitude drops significantly from the unburned to the burned gas side of the flame-brush. Moreover, the vorticity magnitude shows an increasing trend with decreasing *Le*, and increasing turbulence intensity. A significant amount of anisotropy has been observed between the vorticity components within the flame-brush and this anisotropy increases as the wall is approached. The baroclinic torque term has been found to be principally responsible for this anisotropic behaviour. The vortex-stretching and viscous dissipation terms remain the leading order contributors to the vorticity and enstrophy transport for all cases when the flame is away from the wall, but as flame approach the wall, the baroclinic torque begins to play an increasingly important role. The combined molecular diffusion and dissipation contribution to the enstrophy transport remains negative away from the wall but it changes its sign near the wall due to the torque arising from dilatation rate gradient. Detailed physical explanations have been provided for the observed influences of flame and wall on the statistical behaviours of vorticity and enstrophy and the various terms of their transport equations.

1. Introduction

Flame–wall interaction plays a pivotal role in determining the overall energy-efficiency, pollutant formation, as well as the durability and lifespan of combustors in industrial applications. Wall-bounded non-reacting flows have extensively been analysed using Direct Numerical Simulations (DNS) [1,2], but the analysis of flame–wall interaction [3–11] received relatively limited attention. The statistical behaviour of vorticity $\vec{\omega}$ and enstrophy $\Omega = \vec{\omega} \cdot \vec{\omega}/2$ plays an important role in the analysis of turbulent fluid motion [1,2], and these statistics are significantly affected by heat release, density variation and flame normal acceleration in turbulent premixed flames [12]. Several previous analyses focussed on the alignment of $\vec{\omega}$ with local principal strain rates in non-premixed [13–15] and premixed flames [16,17]. These analyses have demonstrated that $\vec{\omega}$ aligns predominantly with the intermediate principal strain rate similar to the non-reacting turbulent flows [18–28], but $\vec{\omega}$ also shows considerable alignment with the most extensive and compressive principal strain rates depending on the relative magnitudes of chemical and turbulent time scales. Chakraborty [17] has revealed that the global Lewis number *Le* (i.e. the ratio between thermal diffusivity α_T and mass diffusivity D) has significant influence on the alignment of $\vec{\omega}$ with local principal strain rates. The analysis by Chakraborty [17] revealed that $\vec{\omega}$ predominantly aligns with the intermediate and the most compressive principal strain rates for low Lewis number flames (e.g. $Le = 0.34$) where the dilatation rate remains almost equal to the most extensive principal strain rate.

The analysis by Hamlington et al. [16] has indicated that enstrophy $\Omega = \vec{\omega} \cdot \vec{\omega}/2$ drops from the unburned to burned gas side of flame-brush. On the contrary, Treurnietet al. [27] reported a localised increase of Ω within the flame-brush [29] for flames with high values of heat release parameter $\tau = (T_{ad} - T_0)/T_0$, where T_0 and T_{ad} are the unburned gas and the adiabatic flame temperatures respectively. Lipatnikov et al. [30] reported both generation and decay of enstrophy across the flame brush in the

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cases of high (e.g. $\tau = 6.53$) and low (e.g. $\tau = 1.5$) values of heat release parameter, respectively. They also analysed the terms of the enstrophy and vorticity transport equation for weakly turbulent premixed flames in the corrugated transport flamelets regime. Recently, Chakraborty et al. [31] have demonstrated that *Le* significantly affects the baroclinic torque contribution to the enstrophy transport and this may lead to an augmentation of Ω within a flame for small values of *Le* under some turbulent flow conditions in the unburned gas, contrary to a decay of Ω across the flame with $Le = 1.0$.

All the aforementioned analyses were conducted in configurations in the absence of walls. However, two-dimensional simulations by Poinsot et al. [3] of head-on quenching (HOQ) and three-dimensional incompressible channel flow DNS of side-wall quenching (SWQ) [4–7,9,10] of turbulent premixed flames revealed that the presence of wall significantly affects vorticity distribution near the wall. However, the statistical behaviours of vorticity $\vec{\omega}$ and enstrophy Ω transport in the presence of wall are yet to be analysed in detail. This paper addresses the aforementioned gap in existing literature by extracting the vorticity $\vec{\omega}$ and enstrophy Ω transport statistics from DNS data of HOQ of statistically planar turbulent premixed flames by an inert isothermal wall for different values of Lewis, Damköhler and Karlovitz numbers. The objectives of this analysis are:

(a) To demonstrate and explain the influences of wall on the statistics of $\vec{\omega}$ and Ω, and the terms of their transport equations,
(b) To identify the influences of *Le* on $\vec{\omega}$ and Ω statistics and their near-wall behaviour.

The rest of the paper will be organised as follows. The mathematical background and numerical implementation pertaining to this analysis are presented next. This will be followed by the presentation of the results and their subsequent discussion. The main findings will be summarised and conclusions will be drawn in the final section of this paper.

2. Mathematical background and numerical implementation

The transport equation of the ith component of vorticity $\omega_i = \varepsilon_{ijk}(\partial u_k/\partial x_j)$ is given by [29–31]:

$$\frac{\partial \omega_i}{\partial t} + u_k\frac{\partial \omega_i}{\partial x_k} = \underbrace{\omega_k\frac{\partial u_i}{\partial x_k}}_{t_{1i}} \underbrace{-\epsilon_{ijk}\frac{1}{\rho^2}\frac{\partial \rho}{\partial x_j}\frac{\partial \tau_{kl}}{\partial x_l}}_{t_{21i}} + \underbrace{\frac{\epsilon_{ijk}}{\rho}\frac{\partial^2 \tau_{kl}}{\partial x_j \partial x_l}}_{t_{22i}} \underbrace{-\omega_i\frac{\partial u_k}{\partial x_k}}_{t_{3i}} + \underbrace{\frac{\epsilon_{ijk}}{\rho^2}\frac{\partial \rho}{\partial x_j}\frac{\partial p}{\partial x_k}}_{t_{4i}} \quad (1)$$

where $\vec{u}$, ρ, p and $\tau_{kl} = \mu(\partial u_k/\partial x_l + \partial u_k/\partial x_l) - 2(\mu/3)\delta_{kl}(\partial u_m/\partial x_m)$ represent the velocity vector, gas density, pressure and stress tensor, respectively with μ being the dynamic viscosity. In Eq. (1), $\vec{t_1}$ is the vortex-stretching term, whereas $\vec{t_{21}}$ and $\vec{t_{22}}$ arise from the misalignment between the gradients of viscous stress and density and from the diffusion of vorticity, respectively. For constant dynamic viscosity (i.e. $\mu =$ constant) incompressible flows, $\vec{t_{22}}$ becomes equal to $(\mu/\rho)\nabla^2\vec{\omega}$. However, for constant dynamic viscosity compressible flows, $\vec{t_{22}}$ takes the form $\vec{t_{22}} = (\mu/\rho)\nabla^2\vec{\omega} + (\mu/3\rho)\nabla\times\nabla(\nabla\cdot\vec{u})$. The term $\vec{t_3}$ accounts for the dilatation contribution, and $\vec{t_4}$ is responsible for the baroclinic effects arising from the misalignment of the density and pressure gradients. Multiplying Eq. (1) by ω_i yields the transport equation for the enstrophy (i.e. $\Omega = \omega_i\omega_i/2$) [30,31]:

$$\frac{\partial \Omega}{\partial t} + u_k\frac{\partial \Omega}{\partial x_k} = \underbrace{\omega_i\omega_k\frac{\partial u_i}{\partial x_k}}_{T_1} \underbrace{-\epsilon_{ijk}\omega_i\frac{1}{\rho^2}\frac{\partial \rho}{\partial x_j}\frac{\partial \tau_{kl}}{\partial x_l}}_{T_2} + \underbrace{\frac{\epsilon_{ijk}\omega_i}{\rho}\frac{\partial^2 \tau_{kl}}{\partial x_j \partial x_l}}_{T_3} \underbrace{-2\frac{\partial u_k}{\partial x_k}\Omega}_{T_4} + \underbrace{\epsilon_{ijk}\frac{\omega_i}{\rho^2}\frac{\partial \rho}{\partial x_j}\frac{\partial \rho}{\partial x_k}}_{T_5}. \quad (2a)$$

The Reynolds-averaged enstrophy (i.e. $\bar{\Omega} = \overline{\omega_i\omega_i}/2$) transport equation can be obtained using Eq. (2a) as [30,31]:

$$\frac{\partial \overline{\Omega}}{\partial t} + \overline{u_k\frac{\partial \Omega}{\partial x_k}} = \underbrace{\overline{\omega_i\omega_k\frac{\partial u_i}{\partial x_k}}}_{T_I} \underbrace{\overline{-\epsilon_{ijk}\omega_i\frac{1}{\rho^2}\frac{\partial \rho}{\partial x_j}\frac{\partial \tau_{kl}}{\partial x_l}}}_{T_{II}} + \underbrace{\overline{\frac{\epsilon_{ijk}\omega_i}{\rho}\frac{\partial^2 \tau_{kl}}{\partial x_j \partial x_l}}}_{T_{III}} \underbrace{\overline{-2\frac{\partial u_k}{\partial x_k}\Omega}}_{T_{IV}} + \underbrace{\overline{\epsilon_{ijk}\frac{\omega_i}{\rho^2}\frac{\partial \rho}{\partial x_j}\frac{\partial \rho}{\partial x_k}}}_{T_V} \quad (2b)$$

where $\tilde{Q}$, $\tilde{Q} = \overline{\rho Q}/\overline{\rho}$ and $Q'' = Q - \tilde{Q}$ are the Reynolds-averaged, Favre-averaged and Favre fluctuation of a general quantity Q respectively. The term T_I indicates the vortex-stretching contribution, whereas T_{II} arises due to misalignment between gradients of density and viscous stresses. The term T_{III} is responsible for molecular diffusion and dissipation of $\overline{\Omega}$, whereas T_{IV} and T_V represent the dilatation and baroclinic torque contributions respectively.

Under the assumption of constant dynamic viscosity, the dissipation rate $\tilde{\epsilon}$ of turbulent kinetic energy $\tilde{k} = \overline{\rho u_i'' u_i''}/2\overline{\rho}$ is closely related to enstrophy as: $\overline{\rho}\tilde{\epsilon} \approx 2\overline{\mu\Omega}$ in the isotropic Kolmogorov turbulence [32]. It has been demonstrated earlier [29,33,34] that the influences of a flame on the major statistical characteristics of a turbulent flow (such as $\tilde{k}$, $\tilde{\epsilon}$ and $\overline{\Omega}$) remain qualitatively similar in the cases of temperature-dependent and constant dynamic viscosity. This similarity stems from the fact that, due to a decrease in the density ρ with increasing temperature, the kinematic viscosity $\nu = \mu/\rho$ increases in flames under both conditions. Here the constant viscosity assumption is adopted for the purpose of simplicity. Under such conditions, the mean dissipation rate $\bar{\epsilon}$ is directly proportional to the mean enstrophy $\overline{\Omega}$. Therefore, the understanding of $\overline{\Omega}$ transport is crucial for addressing the modelling of the dissipation rate of turbulent kinetic energy.

The chemical mechanism is simplified here by a single-step Arrhenius-type mechanism in order to analyse the effects of global Lewis number *Le* in isolation, following several previous analyses [17,38–52]. A single-step Arrhenius-type irreversible chemical reaction is chosen for the current analysis, because three-dimensional DNS simulations with detailed chemistry are extremely expensive for a detailed parametric analysis as carried out in this paper [53]. It is worth indicating that the head-on quenching of premixed flames by isothermal wall is principally driven by heat transfer and not by chemical mechanism [3–7]. Several previous DNS analyses on flame–wall interaction used a single step simplified chemical mechanism [3–7], and the

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