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Effects of anisotropic slip on three-dimensional stagnation-point flow past a permeable moving surface

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ABSTRACT

In this paper, the problem of a steady laminar three-dimensional stagnation point boundary layer flow (Homann stagnation flow) on a permeable moving surface with anisotropic slip in a viscous fluid is investigated. A similarity transformation reduces the governing system of nonlinear partial differential equations into the ordinary (similarity) differential equations. The resulting equations are then solved numerically by using the `bvp4c` function in Matlab. The effects of surface mass transfer parameter, slip parameter and moving parameter on the fluid flow characteristics are presented in the forms of tables and figures and are discussed in detail. Finally, it is concluded from the stability analysis that the first (upper branch) solution is stable and physically realizable, while the second (lower branch) solution is not stable.

1. Introduction

The study of stagnation-point flows has attracted the interest of many researchers because of its applications in industry, including flows over the tips of aircrafts and submarines. Early pioneer works include the two-dimensional stagnation-point flow against a stationary semi-infinite wall by Hiemenz [1], the axisymmetric stagnation flow by Homann [2] and the flow near the stagnation point by Howarth [3]. Stuart [4], Tamada [5] and Dorrepaal [6] investigated the two-dimensional oblique stagnation flow. The stagnation flow close to a saddle point was investigated by Davey [7], while Wang [8] studied the axisymmetric stagnation flow on a circular cylinder.

In addition, the study of viscous flow on a moving wall is also important due to its applications in forced convection cooling processes. The two-dimensional normal stagnation flow towards a plate that is oscillating in its own plane has been studied by Rott [9]. Wang [10] and Libby [11] extended Homann's work by considering the three-dimensional stagnation flow towards a moving plate. Gorla [12] investigated the axisymmetric stagnation flow on a moving circular cylinder. The problem of self-similar boundary layer flow over a moving semi-infinite flat plate is discussed by Weidman et al. [13]. The flows induced by a plate moving normal to planar (Hiemenz) and axisymmetric (Homann) stagnation-point flows has been investigated by Weidman and Sprague [14].

All of the above mentioned studies deal with the no-slip condition on the solid surface. However, such condition is no longer applicable when slip occurs in various situations; for example in perforated plates and wire nettings [15]; lubricated or chemically treated hydrophobic surfaces [16,17]; rough or porous surfaces [18]; and superhydrophobic nano-surfaces [19]. Some examples of industrial applications consisting slip boundary conditions are the polishing of artificial heart valves, rarefied fluid problems, and fluid flow on multiple interfaces. When the no-slip condition is no longer valid, it should be replaced by the slip condition. The linear slip boundary condition was introduced by Navier [20] and Maxwell [21]. A comprehensive discussion and comparison between slip and no-slip boundary condition was prepared by Rao and Rajagopal [22]. Quite a number of studies on stagnation-point flow with a slip boundary condition has been done by Wang [23–25]. The existence and asymptotic behavior of the flow in Wang [23] was investigated by Ishimura and Ushijima [26]. Rosca et al. [27] investigated the steady axisymmetric stagnation point flow and heat transfer due to a permeable moving flat plate with partial slip. Recently, Raees et al. [28] conducted an analysis on three-dimensional stagnation flow of a nanofluid in suspension of both the nanoparticles and microorganisms on moving surface with anisotropic slip.

This present study discusses the effects of anisotropic slip first considered in Wang [25] to three-dimensional stagnation-point

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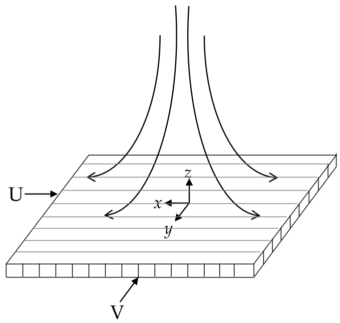

Fig. 1. Physical model and coordinate system when the plate move towards the origin. Notice the x-axis aligns, while y-axis is normal to the plate striations.

flow impinging on a permeable moving plate with different slip coefficients in two orthogonal directions. Anisotropic slip is the type of slip that depends on the flow direction and occurs on geometrically striated surfaces and superhydrophobic strips [29,30]. Following Weidman et al. [13], Weidman and Sprague [14] and Wang [24], a moving parameter ξ has been included into the formulation to study both the cases when the plate is moving out of the origin ($\xi > 0$) and when the plate is moving towards the origin ($\xi < 0$). The system of governing partial differential equations are first transformed into a system of ordinary differential equations, before being solved numerically by using the bvp4c function in Matlab software. The expressions for the skin friction coefficients and velocity profiles are determined to understand the flow characteristics.

2. Mathematical formulation

Consider a steady three-dimensional stagnation point flow of a viscous fluid past a moving permeable plate with anisotropic slip. The Cartesian coordinates x, y and z are measured in the xy-plane, with x- and y-axis are aligned along and normal, respectively, to the striation on the plate, while fluid is placed along the z-axis. The plate is assumed to move out or towards the origin with the constant velocities ξU and ξV in the x and y directions, respectively, where ξ is the dimensionless moving parameter, which is positive ($\xi > 0$) when the plate moves out of the origin and negative ($\xi < 0$) when the plate moves towards the origin, as illustrated in Fig. 1. The constant mass flux velocity is assumed as w_0, with $w_0 < 0$ corresponds to suction and $w_0 > 0$ corresponds to injection or blowing. The velocities of the inviscid (outer) flow are $u_e(x) = ax$, $v_e(y) = ay$ and $w_e(z) = -2az$. Under these assumptions, the governing boundary layer equations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = u_e\frac{\partial u_e}{\partial x} + \nu\nabla^2 u, \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = v_e\frac{\partial v_e}{\partial y} + \nu\nabla^2 v, \tag{3}$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = w_e\frac{\partial w_e}{\partial z} + \nu\nabla^2 w, \tag{4}$$

subject to the boundary conditions

$$\begin{aligned} &u = u_w(x) = \xi U + M_1\mu\frac{\partial u}{\partial z}, \\ &v = v_w(y) = \xi V + M_2\mu\frac{\partial v}{\partial z}, \ w = w_0 \quad \text{at } z = 0, \\ &u = u_e(x) \to ax, \ v = v_e(y) \to ay, \\ &w = w_e(z) \to -2az \quad \text{as } z \to \infty, \end{aligned} \tag{5}$$

where u, v and w are the velocity components along the x-, y- and z-axes, respectively, a is a positive constant, ν is the kinematic viscosity of the fluid, μ is the dynamic viscosity, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator in the Cartesian coordinates (x, y, z), while M_1 and M_2 are the constant slip coefficients in the x- and $y-$directions, respectively.

Introducing the following similarity transformations [25]

$$\begin{aligned} &u = axf'(\eta) + Uh(\eta), \ \ v = ayg'(\eta) + Vk(\eta), \\ &w = -\sqrt{a\nu}[f(\eta) + g(\eta)], \ \ \eta = \sqrt{\frac{a}{\nu}}z, \end{aligned} \tag{6}$$

where primes denote differentiation with respect to η. The normal velocity on the plate is w_0, thus giving $f(0) + g(0) = s$, where $s = -\sqrt{a\nu}$ is the surface mass transfer parameter with $s > 0$ for suction and $s < 0$ for injection, respectively. Without loss of generality, we write $f(0) = s$ and $g(0) = 0$.

By substituting the similarity variables (6) into Eqs. (1)–(4), we find that (1) is automatically satisfied, while (2)–(4) are reduced to the following system of ordinary differential equations

$$f''' + (f + g)f'' + 1 - f'^2 = 0, \tag{7}$$

$$g''' + (f + g)g'' + 1 - g'^2 = 0, \tag{8}$$

$$h'' + (f + g)h' - f'h = 0, \tag{9}$$

$$k'' + (f + g)k' - g'k = 0, \tag{10}$$

and the boundary conditions (5) become

$$\begin{aligned} &f(0) = s, \ \ g(0) = 0, \ \ f'(0) = Af''(0), \ \ g'(0) = Bg''(0), \\ &h(0) = Ah'(0) + \xi, \ \ k(0) = Bk'(0) + \xi, \\ &f'(\eta) \to 1, \ \ g'(\eta) \to 1, \ \ h(\eta) \to 0, \ \ k(\eta) \to 0 \quad \text{as } \eta \to \infty, \end{aligned} \tag{11}$$

where $A = M_1\mu\sqrt{a/\nu}$ and $B = M_2\mu\sqrt{a/\nu}$ are the dimensionless slip parameters. For the case of impermeable plate ($s = 0$) which moves away from the origin ($\lambda = 1$), Eqs. (7)–(11) are similar with those of Wang [25]. We also notice that the two-dimensional case of the moving plate can been recovered when $g = h = k = 0$ and $\lambda < 0$, while the axisymmetric case can be recovered when $f = g$ and $\lambda > 0$.

The physical quantities of practical interest are the skin frictions τ_{wx} and τ_{wy} in the x- and y-directions of the moving plate, which are defined as

$$\begin{aligned} &\tau_{wx} = M_1\mu\left(\frac{\partial u}{\partial z}\right)_{z=0} = A[axf''(0) + Uh'(0)], \\ &\tau_{wy} = M_2\mu\left(\frac{\partial v}{\partial z}\right)_{z=0} = B[ayg''(0) + Vk'(0)]. \end{aligned} \tag{12}$$

3. Stability analysis

We test the fact that the dual solutions of the problem (7)–(11) are stable or unstable by performing a stability analysis. In this respect, we follow Weidman et al. [13] and Rosca and Pop [31,32] who have shown for the forced convection boundary layer flow past a permeable flat plate and, respectively, for the mixed convection flow past a vertical flat plate, that the lower branch solutions are unstable (not realizable physically), while the upper branch solutions are stable (physically realizable). We begin the analysis by considering the unsteady state for boundary value problem (1)–(4). Eq. (1) holds, while (2)–(4) become

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = u_e\frac{\partial u_e}{\partial x} + \nu\nabla^2 u, \tag{13}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = v_e\frac{\partial v_e}{\partial y} + \nu\nabla^2 v, \tag{14}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = w_e\frac{\partial w_e}{\partial z} + \nu\nabla^2 w. \tag{15}$$