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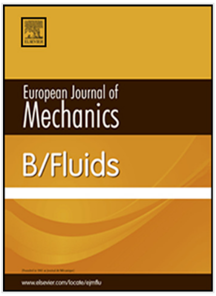

Competing instabilities of rotating boundary-layer flows in an axial free-stream

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Competing instabilities of rotating boundary-layer flows in an axial free-stream

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Abstract

In this study, a new centrifugal instability mode, which dominates within the boundary-layer flow over a slender rotating cone, defined by half-angle $\psi < 40°$, is used for the first time to model the problem when an enforced oncoming axial flow is introduced. The resulting similarity solution represents the basic flow more accurately than previous studies in the literature. This mean flow field is subsequently perturbed leading to disturbance equations that are solved via numerical and analytical approaches, importantly yielding favourable comparison with existing experiments. Meanwhile, a formulation consistent with the classic rotating-disk problem has been successful in predicting the stability characteristics of broad rotating cones, defined by half-angle $\psi > 40°$, in axial flow.

Keywords

Rotating boundary-layer — crossflow instability — centrifugal instability — broad/slender rotating cone — co-rotating/counter-rotating spiral vortices

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1. INTRODUCTION

This article forms part of a series of studies which have used theoretical techniques to construct the correct models of governing instability for both broad and slender rotating cones. The current study represents a significant extension to the general problem in the slender cone case, introduced when enforcing an oncoming axial flow.

Physically, the problem represents an accurate model of axial airflow over rotating machinery components at the leading edge of a turbofan. In such applications, laminar-turbulent transition within the boundary layer can lead to significant increases in drag, resulting in negative implications for fuel efficiency, energy consumption and noise generation. Consequently, delaying transition to turbulent flow is seen as beneficial, and controlling the primary instability may be one route to achieving this. Ultimately, control of the input parameters of such a problem may lead to future design modifications and potential cost savings.

Our results are discussed in terms of existing experimental data and previous stability analyses on related bodies. Importantly, axial flow is seen to delay the onset of convective instability for both broad and slender rotating cones; the exact mechanism of interaction governing the transition process however is very different for both instabilities. Broad-angled rotating cones are susceptible to a crossflow instability visualised in terms of co-rotating spiral vortices, whereas slender rotating cones have transition characteristics governed by a centrifugal instability, which is visualised by the appearance of counter-rotating Görtler vortices. It is the relative competition of these two governing mechanisms that is explored in detail in this study, particularly with regard to the role of travelling modes in the breakdown process.

2. METHODS

We consider a cone of half-angle ψ rotating in a fluid of kinematic viscosity ν^* with an angular velocity Ω^* in an anti-clockwise direction around the streamwise coordinate axis x^* (where a * denotes a dimensional quantity in all that follows). We construct coordinate axes aligned along with and perpendicular to the spiral vortices ($\hat{x}^*$ and y^*, respectively), as shown in Figure 1. Further details of the relationship between the coordinate systems, including a required Mangler transformation, are provided in [5]. These are shifted from the conventional streamwise and azimuthal coordinates, x^* and θ, which are based on cylindrical polar coordinates. In such a problem, there exists a boundary layer close to the rotating cone surface characterised by the distance along the cone l^* and defined by the Reynolds number, R, such that:

$$\mathrm{R} = \frac{\Omega^* l^{*2} \sin\psi}{\nu^*}.$$

With the important distinction of the inclusion of the oncoming axial flow, the physical problem is subsequently altered such that there now exists a dimensional local slip velocity at the edge of the boundary layer, obtained via a well-known potential-flow solution (see for example [1]), given by $U_e = C^* x^{*m}$, where C^* is a constant.

We subsequently compare this velocity to the rotational velocity of the cone surface, given by $V_w = \Omega^* x^* \sin\psi$, to