European Journal of Mechanics B/Fluids 63 (2017) 66–72

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Energy description for deforming bodies moving through inviscid fluids

Ren Sun *, Yi Qin

Department of Engineering Mechanics, Shanghai Jiao Tong University, Shanghai 200240, China

ARTICLE INFO

Article history:
Received 24 May 2016
Received in revised form
3 November 2016
Accepted 16 January 2017
Available online 24 January 2017

Keywords:
Multiple deformable bodies
Potential flow
Unsteady description

ABSTRACT

The energy-based Lagrangian expressions for multiple deforming bodies translating and rotating through an inviscid fluid in the gravity field are proved to be equal to the momentum-type ones This implies that the energy description can be directly adopted to predict dynamic behaviors of these deforming bodies without adding the body deformation thrust to fluid as the generalized force of the body–fluid system although the total mechanical energy of the system is continuously changed by the deformation effect And it is identified that the hydrodynamic force or torque on any single deforming body includes contributions of linear and angular accelerations of all the bodies, which means that these acceleration constituents should be added to corresponding hydrodynamic loads so as to unsteadily describe motions of the deforming body. In corresponding two dimensional cases, the energy description still holds and may be transformed into the extended unsteady Blasius formulae. Therefore, one can choose any pertinent approach to deal with hydrodynamic interactions among many deformable bodies.

1. Introduction

In recent years there has been a revival of interest in investigation of the dynamics of many bodies moving through an inviscid liquid. The hydrodynamic interaction between these bodies is of fundamental importance to unsteady description of their subsequent motions.

Theoretically, there are two approaches used to describe the hydrodynamic interactions among multiple rigid bodies in potential flows. One is based on a momentum-type description, in which the hydrodynamic loads are derived by integrating the pressure over each body surface [1–3]. Usually, the pressure is expressed in terms of the local rate of change of the velocity potential and the squared flow velocity as per the Bernoulli equation so as to avoid the direct computation of the pressure distribution, but integrations of them are very cumbersome in coping with cases of many rigid bodies [4,5]. The approach has been adopted to resolve problems of hydrodynamic couplings among two rigid or deformable bodies [6,7]. The other is concerned with the energy description [1,2], which applies the Hamilton variational principle to an energy-conservative body–fluid system. This means has been extended to description of many moving rigid bodies through an inviscid fluid [8–14]. It is worthwhile to note that it can clearly express contributions of linear and angular accelerations of all the bodies to the hydrodynamic force or torque on any single body, and these shares are needed for the unsteady description of motions of velocity-changing bodies. In two-dimensional problems, the extended unsteady Blasius formulae can be adopted as a convenient tool to predict dynamical behaviors of multiple moving cylinders [1,7,15,16].

The energy approach in analysis has advantages that there is no need to integrate the squared flow velocity over each body surface and the time derivative is outside an integral of the velocity potential, so it is suitable for evaluating hydrodynamic interactions among several moving rigid bodies [14]. However, it would be found from references that this powerful tool has not yet been used to deal with those hydrodynamic couplings with deformable bodies. There may be some doubt that whether thrusts due to the body deformation to fluid should be added as generalized forces to the Lagrangian expressions or not when deforming bodies are moving through an inviscid fluid, providing continuously an extra share of energy to the flow field from their own deformations, and thus the total mechanical energy of the body–fluid system cannot maintain unchanged. In addition, one would query the validity of the energy description in the corresponding two-dimensional cases because the kinetic energy of the fluid is unintegrable. In order to make use of the energy description as an applicable tool for investigating swimming of deforming bodies through an inviscid fluid, this paper will prove that for all two-dimensional

* Corresponding author.
E-mail address: drrsun@sjtu.edu.cn (R. Sun).

http://dx.doi.org/10.1016/j.euromechflu.2017.01.009

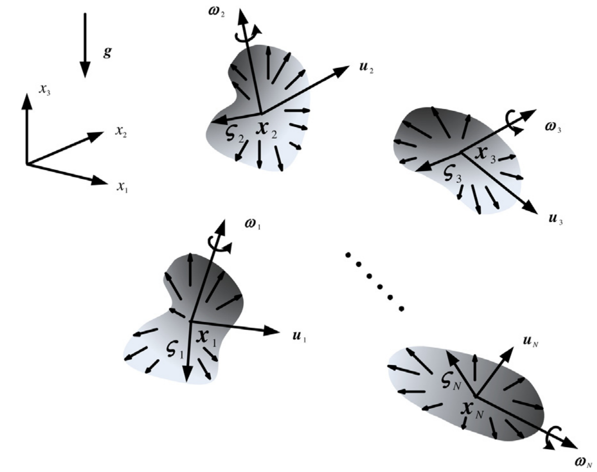

Fig. 1. Sketch of many deforming bodies moving through an inviscid fluid.

and three-dimensional cases, the energy approach is equivalent to the corresponding momentum-type one without considering bodies' deformation thrusts as the generalized forces expressed in the Lagrangian expressions.

2. Statement of the problem

Let us consider such motions of N arbitrarily-shaped deforming bodies through an otherwise quiescent unbounded fluid in the field of gravity acceleration $\boldsymbol{g}$. In an absolute Cartesian coordinate system (x_1, x_2, x_3), body β $(\beta \in \{1, \ldots, N\})$, with its centroid located at $\boldsymbol{x}_\beta = x_{\beta i}\boldsymbol{e}_i$, moves at a translational velocity $\boldsymbol{u}_\beta = \boldsymbol{u}_{\beta i}\boldsymbol{e}_i$ and an angular velocity $\boldsymbol{\omega}_\beta = \boldsymbol{\omega}_{\beta i}\boldsymbol{e}_i$ around the instantaneous axis through the centroid, where a repeated index indicates a summation except for β or γ, $i = 1, 2, 3$, $\boldsymbol{e}_i$ denotes a unit vector parallel to axis x_i, $x_{\beta i}$ the ith coordinate of the centroid, and $u_{\beta i}$ and $\omega_{\beta i}$ its ith translational velocity component and angular velocity component, respectively. Moreover, the orientation of the body is denoted by three unit vectors $\boldsymbol{e}_{\beta j}$ $(j = 1, 2, 3)$ of a moving coordinate system fixed at the centroid since a series of arbitrary rotations of a body cannot be equivalent to a single rotation but to three Euler rotations. The infinitesimal angular displacement vector of the body is expressed as $\delta\boldsymbol{\theta}_\beta = \delta\theta_{\beta i}\boldsymbol{e}_i$, and the radial vector from the centroid to its surface is written as $\boldsymbol{\varsigma}_\beta = \varsigma_{\beta i}\boldsymbol{e}_i$ changeable with instantaneous velocity $\boldsymbol{u}_{d\beta} = u_{d\beta i}\boldsymbol{e}_i$. A kinematic configuration of these moving bodies in a fluid is plotted in Fig. 1.

On the assumption that the fluid is inviscid and incompressible, and the flow irrotational, there exists a velocity potential $\phi = \phi\,(\boldsymbol{x}, \boldsymbol{x}_\lambda, \boldsymbol{u}_\lambda, \boldsymbol{e}_{\lambda i}, \omega_\lambda;\, \boldsymbol{\varsigma}_\lambda, \boldsymbol{u}_{d\lambda} | \lambda = 1, \ldots, N; i = 1, 2, 3)$ at any position $\boldsymbol{x}(x_1, x_2, x_3)$ in the flow field, which can be expressed as (cf. [17])

$$\phi = \boldsymbol{u}_\alpha \bullet \boldsymbol{\varphi}_\alpha + \boldsymbol{\omega}_\alpha \bullet \boldsymbol{\varphi}_\alpha^* + \phi_d \quad (\alpha = 1, \ldots, N), \tag{1}$$

where $\boldsymbol{\varphi}_\alpha$ and $\boldsymbol{\varphi}_\alpha^*$ denote unit velocity potential vectors respectively associated with the translation and rotation of body α, and ϕ_d is the additional potential associated with the deformations of the bodies. The boundary condition on the surface S_β of body β takes in the alternative forms

$$\frac{\partial\phi}{\partial n_\beta} = \boldsymbol{n}_\beta \bullet \nabla\phi = \boldsymbol{u}_\beta \bullet \boldsymbol{n}_\beta + \boldsymbol{\omega}_\beta \bullet (\boldsymbol{\varsigma}_\beta \times \boldsymbol{n}_\beta) + \boldsymbol{u}_{d\beta} \bullet \boldsymbol{n}_\beta \tag{2a}$$

or

$$\frac{\partial\boldsymbol{\varphi}_\alpha}{\partial n_\beta} = \delta_{\alpha\beta}\boldsymbol{n}_\beta, \qquad \frac{\partial\boldsymbol{\varphi}_\alpha^*}{\partial n_\beta} = \delta_{\alpha\beta}\boldsymbol{\varsigma}_\beta \times \boldsymbol{n}_\beta, \qquad \frac{\partial\phi_d}{\partial n_\beta} = \boldsymbol{u}_{d\beta} \bullet \boldsymbol{n}_\beta, \tag{2b}$$

where $\boldsymbol{n}_\beta = n_{\beta i}\boldsymbol{e}_i$ denotes a unit outward normal vector to the body surface, n_β distance along the normal, $\delta_{\alpha\beta}$ the Kronecker delta.

2.1. Hydrodynamic loads in the momentum-type description

The hydrodynamic force $\boldsymbol{F}_\beta$ exerted on body β by the surrounding fluid is expressed integrating the pressure p over its surface as

$$\boldsymbol{F}_\beta = -\int_{S_\beta} p\,\boldsymbol{n}_\beta\, dS. \tag{3}$$

Since the flow is potential and incompressible, the pressure at an arbitrary point in the flow field obeys the Bernoulli equation,

$$\frac{p}{\rho} = -\frac{\partial\phi}{\partial t} - \frac{\nabla\phi \bullet \nabla\phi}{2} + \boldsymbol{g} \bullet \boldsymbol{x} + const., \tag{4}$$

where ρ denotes the fluid density. $\boldsymbol{F}_\beta$ is then rewritten as

$$\boldsymbol{F}_\beta = \int_{S_\beta} \rho\,\frac{\partial\phi}{\partial t}\,\boldsymbol{n}_\beta dS + \frac{\rho}{2}\int_{S_\beta} (\nabla\phi \bullet \nabla\phi)\,\boldsymbol{n}_\beta dS - \rho V_\beta \boldsymbol{g}, \tag{5a}$$

where V_β denotes the instantaneous volume of deforming body β. Similarly, the expression for torque $\boldsymbol{T}_\beta$ acting on the body about its centroid becomes

$$\begin{aligned} \boldsymbol{T}_\beta &= -\int_{S_\beta} p\,\boldsymbol{\varsigma}_\beta \times \boldsymbol{n}_\beta\, dS \\ &= \int_{S_\beta} \rho\,\frac{\partial\phi}{\partial t}\,\boldsymbol{\varsigma}_\beta \times \boldsymbol{n}_\beta dS + \frac{\rho}{2}\int_{S_\beta} (\nabla\phi \bullet \nabla\phi)\,\boldsymbol{\varsigma}_\beta \\ &\quad \times \boldsymbol{n}_\beta dS + \rho V_\beta \boldsymbol{g} \times \bar{\boldsymbol{x}}_\beta, \end{aligned} \tag{5b}$$

where $\bar{\boldsymbol{x}}_\beta = \boldsymbol{x}_\beta^\dagger - \boldsymbol{x}_\beta$ is a position vector of the center $\boldsymbol{x}_\beta^\dagger$ of buoyancy of body β relative to its centroid.

2.2. Hydrodynamic loads in the Lagrangian description

On the above assumption, the kinetic energy T_f of fluid may be expressed as a surface integral,

$$T_f = -\frac{\rho}{2}\int_{\Sigma S_\gamma} \phi\,\frac{\partial\phi}{\partial n_\gamma}\, dS, \tag{6}$$

where ΣS_γ indicates the sum of N body surfaces. Substitution of (1) and (2) into (6) results in

$$\begin{aligned} T_f &= -\frac{\rho}{2}\int_{\Sigma S_\gamma} (\boldsymbol{u}_\alpha \bullet \boldsymbol{\varphi}_\alpha + \boldsymbol{\omega}_\alpha \bullet \boldsymbol{\varphi}_\alpha^* + \phi_d)(\delta_{\lambda\gamma}\boldsymbol{u}_\lambda \bullet \boldsymbol{n}_\gamma \\ &\quad + \delta_{\lambda\gamma}\boldsymbol{\omega}_\lambda \bullet \boldsymbol{\varsigma}_\gamma \times \boldsymbol{n}_\gamma + \boldsymbol{u}_{d\gamma} \bullet \boldsymbol{n}_\gamma)\, dS \\ &\quad (\alpha, \lambda = 1, \ldots, N). \end{aligned} \tag{7}$$

And the gravitational potential energy U_f of the fluid with N void volumes occupied by the N bodies can be written as

$$U_f = \rho V_\alpha \boldsymbol{g} \bullet \boldsymbol{x}_\alpha^\dagger + const. \quad (\alpha = 1, \ldots, N), \tag{8}$$

that is, U_f increases as the void volumes migrate along the sense of the gravity acceleration.

If Lagrangian function of the fluid is expressed by $L = T_f - U_f$, the Lagrange equations of motion give a hydrodynamic force acting on body β

$$\boldsymbol{F}_\beta = -\frac{d}{dt}\left(\frac{\partial L}{\partial \boldsymbol{u}_\beta}\right) + \frac{\partial L}{\partial \boldsymbol{x}_\beta} \tag{9a}$$