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Macroscopic limit for an evaporation–condensation problem

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ABSTRACT

We investigate the hydrodynamic limit of a vapor–noncondensable gas mixture in a pressure gradient between two walls. Earlier papers based on conventional asymptotic analysis techniques predict the emergence of a boundary layer of noncondensables which completely blocks the vapor flow (Takata and Aoki, 2001; Aoki et al., 2003). This "ghost effect" (Sone, 2007) contradicts physical intuition. In the present paper we reveal the bifurcation structure of the underlying transport operator and combine it with an appropriate macroscopic scaling. As a result, the hydrodynamic limit describes the coexistence of a streaming mode of vapor with the other component at rest thus avoiding the ghost effect.

For sake of clarity, the paper restricts to a simplified setting (discrete velocity model, mechanically identical particles). However, the results also apply in more general situations.

1. Introduction

The paper deals with the kinetic modeling of gas mixtures in the fluid dynamic limit. Both problems, the kinetics of mixtures (and in particular: vapor–noncondensable mixtures) and questions concerning the fluid dynamic limit of rarefied flows, have been recently investigated in a couple of papers (see, e.g. [1–3] for the first, and [2,4] with the literature cited there for the latter aspects). One particular problem is that of boundary layers for mixtures. Due to the scaling used at small Mach numbers, it seems sufficient to study planar problems like half space or slab geometries. Once this is completely understood (see [1] for the evaporation–condensation problem discussed below, and [3] for the discrete velocity case), such boundary layers can be used to couple boundaries to hydrodynamic flow fields.

For the fluid dynamic limit there exist well-established and widely accepted techniques like the Chapman–Enskog or the Hilbert expansion. Matching these flow fields to boundary layers designed e.g. to adapt the flow to diffusive reflection laws, seems to create sufficiently good solutions in a number of problems. The situation changes if a local matching of the boundary or interface conditions is not sufficient, since they serve as global control mechanisms. Such a situation occurs in the case of a binary gas mixture consisting of vapor and of a noncondensable between parallel walls. In the case of a pressure difference between the walls, vapor starts to move from one wall (evaporation) to the other (condensation) thus introducing a flux. The noncondensable follows this flux and creates a barrier at one of the walls slowing down the vapor flux. Following the classical tools from asymptotic analysis, this barrier has to be modeled and matched as a boundary layer. This leads to a result in the hydrodynamic limit which is in literature known as a "ghost effect" and which obviously does not yield an appropriate description of the hydrodynamic limit.

The aim of the paper is to provide an alternative description to this situation. Based on a detailed view on the collision operator and its algebraic structure, and combined with an appropriate scaling it yields a solution to the evaporation–condensation problem which is much more intuitive from a physical point of view. Due to the lack of data possibly used for comparisons and benchmarks (experimental data like those in [5,6] are not applicable; reliable numerical results are very hard to obtain due to stiffness problems) it is a question of plausibility whether to accept the presented ansatz or to look for an alternative. In our opinion, it provides a reliable perspective, since the bifurcation property (which is the keystone of the investigations) is based on a well-known property of the algebraic structure of the involved operator (this property is not worked out in investigations like those in [3], which are thus restricted to small perturbations), and the choice of diffusive scaling has proven useful in a variety of problems for the derivation of diffusion phenomena [7–9]. This scaling gets rid of the ghost effect; furthermore it also seems to be the appropriate way to solve a couple of other problems related to the connections of the Navier–Stokes and the Boltzmann equations (like the problem discussed in [10]).

Consider a gas mixture composed of two species confined between two parallel walls (Fig. 1). Species A ("vapor") is emitted and adsorbed at the boundaries according to a prescribed pressure

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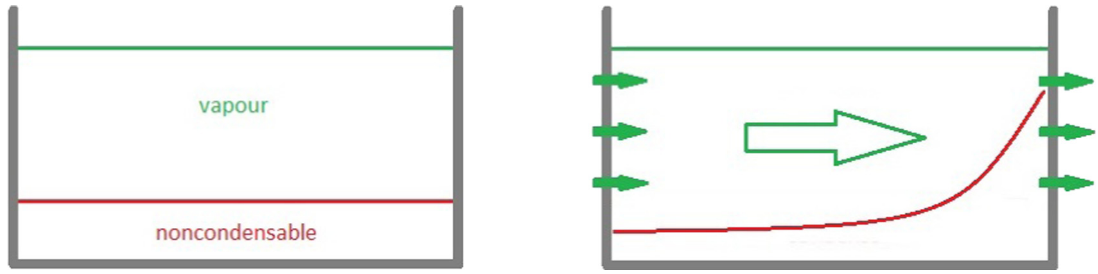

Fig. 1. Schematic view on density profiles for mixture (upper line) and noncondensable (lower line) without (left) and with vapor flux (right).

difference. Species B ("noncondensable") is totally reflected when hitting the walls. If there is a pressure decay from the wall at $x = a$ to the wall at $x = b$, then a flow of vapor is induced from a to b. At the same time one expects the noncondensable to follow the flow and form a boundary layer at b which slows down the vapor flow.

This problem has been studied in a couple of papers in recent years and in particular the fluid dynamic limit was of interest (see, e.g. [11,12]). It turned out that the application of standard asymptotic analysis methods for the fluid dynamic limit leads to a curious situation. In the limit both A and B exhibit the same Maxwellian profile with an infinitesimally small bulk velocity $\overline{v}$ (in fact, $\overline{v} = 0$), and a thin boundary layer of noncondensable is formed at b completely suppressing the vapor flow. This phenomenon contradicts physical intuition and is known in the literature as *ghost effect* [13].

In the present work we propose a different macroscopic limit, based on a scaling ("diffusive scaling") which in the past has been applied in a variety of problems for the derivation of diffusion phenomena (see, e.g. [7–9]). It turns out that this kind of scaling leads in the limit to a boundary layer of well-defined thickness for the noncondensable which slows down but does not stop the vapor flow. The results are based on a careful investigation of a bifurcation phenomenon of the governing transport operator in the presence of a small drift. In case of zero drift, its nullspace has geometric dimension one (related to mass flow conservation) but algebraic multiplicity two. At the emergence of a drift, the two-dimensional nullspace splits up into two simple eigenspaces giving rise to a new nonzero eigenvalue.

In this paper we are not interested at all in any existence theorem for some boundary value problems. The only intention is to provide a scenario which allows in the hydrodynamic limit a gas mixture with one component at rest while the second one is streaming. This is the main result, and it reveals the major difference to the classical asymptotic expansions for which such a result is not possible (see [11,12]). In order not to hide this behind a bunch of technical details, we restrict to the following simplified setting. However, due to the similarities of the structure of the underlying transport operators (see, e.g. [14,15] in the case of the continuous and the discrete Boltzmann collision operator), we are convinced, that the present results can be generalized.

We investigate the problem in the framework of Discrete Velocity Models (DVM) on the basis of two-particle collisions (see [16]). We consider the steady spatially one-dimensional problem in the slab [0, 1] in the simplest possible case of mechanically identical species A and B. This means that both are driven by the same Boltzmann collision operator. The only difference is the wall interaction. Denote by $\mathbf{g}$ the distribution of A and by $\mathbf{h}$ that of B. Then the sum $\mathbf{f} = \mathbf{g} + \mathbf{h}$ is governed by a nonlinear one-species Boltzmann collision operator J. If $\mathbf{f}$ is known, then $\mathbf{g}$ and $\mathbf{h}$ evolve according to a *linear* transport operator. We restrict to the case of $\mathbf{f}$ being a fixed global Maxwellian. In the case of zero flow between 0 and 1, $\mathbf{f}$ is a centered Maxwellian with zero bulk velocity. The corresponding transport operator L_0 exhibits a typical structure concerning the algebraic nullspace which in a similar situation has been observed in a couple of papers (e.g. [14] for the continuum case, [17,15] for DVM).

For our investigation we require the DVM to satisfy four assumptions (see (2.3), (2.4), (3.2), (3.8) below), two of them being crucial. The first one is a symmetry condition and requires the velocity grid and the collision model to be invariant under a change of sign of the velocity components perpendicular to the walls. This leads to a linear ODE system with a matrix having a special anti-symmetric block structure which is essential. (In the paper we exclude the case of zero normal velocities which would lead to a DAE rather than an ODE system. However, numerical experiments indicate that this condition can be weakened.) The second assumption concerns the existence of a maximal number of pairwise different nonzero eigenvectors. This in particular prohibits the existence of artificial invariants of the transport operator. (A discussion of this point may be found in [17,15].)

2. The evaporation–condensation problem

2.1. The model

Consider a gas mixture confined in the slab [0, 1]. The two components of the gas are species A ("vapor") with density function $\mathbf{g}(t, x, \mathbf{v})$ and species B ("noncondensable") with density function $\mathbf{h}(t, x, \mathbf{v})$. The are represented in the form $\mathbf{v} = (v_x, v_\perp)$, with v_x the component pointing in x-direction, and $v_\perp$ the orthogonal complement.

Concerning the gas particle interaction, both types are mechanically identical in the sense that both are governed by the same Boltzmann collision operator. The only difference lies in the gas–wall interaction. While species A may pass through the walls in both directions (condensation, evaporation), species B is totally reflected. As a consequence, there may be a total nonzero mass flux of A through the walls while the mass flux of B is zero.

We write $\mathbf{f} = \mathbf{g} + \mathbf{h}$ and let the governing equations for $\mathbf{g}$ and $\mathbf{h}$ be the nonlinear two-species Boltzmann equation

$$(\partial_t + v_x \partial_x)\mathbf{g} = J[\mathbf{f}, \mathbf{g}] \tag{2.1}$$

$$(\partial_t + v_x \partial_x)\mathbf{h} = J[\mathbf{f}, \mathbf{h}] \tag{2.2}$$

with the collision operator $J[., .]$ to be specified below. Since $J[., .]$ is bilinear, a consequence of (2.1), (2.2) is that $\mathbf{f}$ solves the nonlinear Boltzmann equation

$$(\partial_t + v_x \partial_x)\mathbf{f} = J[\mathbf{f}, \mathbf{f}]. \tag{2.3}$$

In most of the paper we restrict to the steady variant of the system,

$$v_x \partial_x \mathbf{g} = J[\mathbf{f}, \mathbf{g}] \tag{2.4}$$

$$v_x \partial_x \mathbf{h} = J[\mathbf{f}, \mathbf{h}]. \tag{2.5}$$

In order to extend the equations to a well-posed boundary value problem, they have to be supplemented with boundary conditions either in the form of reflection laws or by prescribing the flows into the domain [0, 1]. For our purposes such a detailed description is not necessary.