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Thin body motion in compressible fluid in the presence of free and rigid boundaries

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ABSTRACT

Two-dimensional problems of thin body motion in fluid parallel to the boundary at a distance comparable with the length of the body are regarded. In particular, the resistance and lift forces in motion parallel to free and rigid surfaces are determined. The fluid is assumed to occupy infinite semi-space and gravity is neglected, in comparison with the fluid inertia. The solution is obtained for the problem of plate motion in slightly compressible fluid at a definite depth under a free surface, with constant velocity and inclination angle. The formation of a finite-length cavity behind the body is taken into account. The solution for the linearized problem of a profile motion near a flat, solid surface in a slightly compressible fluid is given. The screen effect becomes essential when the altitude is smaller than the chord. The numerical solution for the incompressible flow about a profile flying over a rigid surface has been obtained by using a panel method. In this way, the numerical lift is compared with the analytical solution. Moreover, streamlines and velocities are discussed by placing the profile at different altitudes and incidences.

1. Introduction

In the present paper linear, approximate analyses of the flows about a profile moving below a free surface and flying above a solid wall are discussed.

The first investigation has many practical applications, such as determining the resistance and lift forces as functions of the depth in underwater motion of a bullet or shell, which is important for evaluating the consequences of the puncture of orbit liquid filled tanks by small space debris [1,2]. Similar problems arise in evaluating the effects of wave-breakers on the streaming flow. The problem is relevant to surface or underwater high-velocity gliding of a thin profile, which is often used to reduce the resistance of a glider. Many gliding problems have been solved for the case of incompressible fluid, because flow velocities were expected to be rather low.

The problem of gliding near a free surface has been considered within the framework of linear [3–7] and nonlinear [8–10] formulations and found its generalized classical solution in the book by Sedov [11]. The solution for the plate with a large angle of attack was obtained by Karlikov and Tolokonnikov [12], for the case of an unbounded incompressible fluid. High-speed streaming flows accounting fluid compressibility were investigated in [13–15]. The problems studied in [16] include the cases of both positive and negative angles of attack. However, the solutions relied heavily on the fact that the cavity behind the gliding body has an infinite length. The presence of a finite-length cavity has been considered in [16].

At the beginning of the 20th century it was observed that the lift force of a wing moving near a flat surface increases strongly in comparison with free flight. This fact was used in the creation of new flying devices (screen-flights), which got the Russian name "*ekranoplan*". The analytical solution of linear problems for the profile motion near a flat surface is very important for verifying the numerical modelling of this flow. Sedov obtained an analytical solution for the lift force of a profile moving near a rigid surface in terms of Weierstrass functions [11], by introducing the theory of a complex variable. Unfortunately, this solution includes free constants that cannot be easily evaluated. An approximate

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analytical solution for the problem of non-steady plane motion near rigid surface was obtained by Rozhdestvensky [17]. A theoretical investigation of the wing motion near a rigid surface was made by Panchenkov [18,19], and experimental results have been discussed in [20].

The present paper is organized as follows. Some relevant statements about the two problems are discussed in Section 2. The flow generated by the motion of a profile below a free surface is analysed in Section 3. The evaluation of the pressure inside the cavity for a given cavity length leads to a solution free from undetermined constants. Also the inverse calculation (evaluating the size of the cavity by starting from its pressure) is possible, even if it implies the (numerical) solution of a transcendental equation. The analysis of the flow around a profile flying above a solid wall is presented in Section 4. In Section 5, the force on the profile given by the analytical solution is compared with the one obtained from a numerical simulation of the flow with a panel method. Finally, some concluding remarks are presented in Section 6.

2. Mathematical statement of the problems

In the present paper, two kinds of steady, planar and irrotational flows of a slightly compressible, inviscid fluid are investigated in an analytical fashion, by using linear approximations. The first one is the flow generated by the motion of a profile just below a flat free surface, in the presence of a cavity behind the body. The second flow is due to a profile flying at small distances above a solid, flat wall.

As the fluid is slightly compressible, the Mach number M_0 referred to the asymptotic velocity (V_0) is very small. In this condition, once the quantity $\delta := (1 - M_0^2)^{1/2}$ and the new coordinate[1] $\tilde{x} := x/\delta$ have been introduced, the disturbance velocity potential φ satisfies the Laplace equation: $\partial^2_{\tilde{x}\tilde{x}}\varphi + \partial^2_{yy}\varphi = 0$ and the pressure p is governed by the linearized Bernoulli equation:

$$p + \frac{\rho_0 V_0}{\delta}\partial_{\tilde{x}}\varphi \equiv p_0, \qquad (1)$$

with p_0 the ambient pressure at large distances from the profile.

The flow around a profile moving at depth h below a free surface is investigated in Section 3. The free surface is assumed flat and is aligned along the x-axis, while the y-axis is directed inside the liquid. On this boundary of the liquid domain, the pressure (p) is assumed uniform and equal to the ambient pressure (p_0), *i.e.* $p \equiv p_0$ or, by means of Bernoulli equation (1), $\partial_{\tilde{x}}\varphi = 0$. The profile is defined by the two functions $h^{\pm}(\tilde{x})$, giving the positions of two points on upper (h^+) and lower (h^-) sides of the boundary, corresponding to the abscissa $\tilde{x} \in [0, \tilde{L}]$, $\tilde{L}$ being the chord of the profile L divided by δ. The local slope angles $\alpha^{\pm}$ on the upper and lower sides of the profile are calculated as $\alpha^{\pm} = +dh^{\pm}/dx$. The presence of a cavity behind the profile is accounted for. Inside the cavity there is a mixture of gas and vapour at a pressure p_c slightly lower than the ambient one. The difference $p_0 - p_c$ will be named as Δp hereafter. In the linearized framework, the boundary condition on the body is enforced on the straight segment $\boldsymbol{z} = \boldsymbol{i}h^+ + \sigma$ (bold characters indicate complex quantities, $\boldsymbol{i}$ is the imaginary unit) with $0 < \sigma < \tilde{L}$. The condition is that the normal component of the velocity vanishes: $\partial_y\varphi = V_0\alpha^+$. In the linear approximation, the cavity is pushed on the straight segment $\boldsymbol{z} = \boldsymbol{i}h^- + \sigma$ with

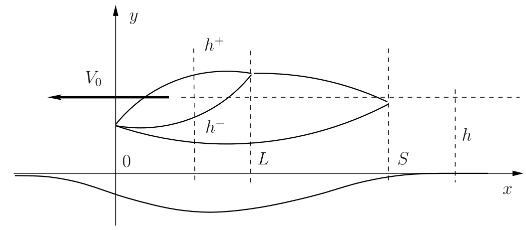

Fig. 1. Sketch of the flowfield for the motion with a free surface.

$0 < \sigma < \tilde{S}$, on which the boundary condition $p = p_c$ is enforced, or, by means of the Bernoulli equation (1), $\partial_{\tilde{x}}\varphi = \delta\Delta p/(\rho_0 V_0)$. On the remaining part of the straight line $\boldsymbol{z} = \boldsymbol{i}h + \sigma$ (for $\sigma > S$) it is enforced that the pressure recovers its ambient value, so that the Bernoulli equation (1) implies: $\partial_{\tilde{x}}\varphi = 0$. In summary, the boundary conditions are:

$$\begin{cases} 0 < \tilde{x} < \tilde{L}, \quad y = h^+ : \partial_y\varphi = +V_0\alpha^+, \\ \quad y = h^- : \partial_{\tilde{x}}\varphi = \dfrac{\delta\Delta p}{\rho_0 V_0} \\ \tilde{L} < \tilde{x} < \tilde{S}, \quad y = h : \partial_{\tilde{x}}\varphi = \dfrac{\delta\Delta p}{\rho_0 V_0} \\ \tilde{x} > \tilde{S}, \quad y = h : \partial_{\tilde{x}}\varphi = 0 \\ -\infty < \tilde{x} < +\infty, \quad y = 0 : \partial_{\tilde{x}}\varphi = 0. \end{cases} \qquad (2)$$

In Section 4 the flow around a profile flying over a flat wall is investigated. The profile is defined as before, but the local slope angles are now defined in the usual aeronautical way: $\alpha^{\pm} := -dh^{\pm}/dx$. In the linear framework, the boundary conditions of vanishing normal velocities are enforced on the segments $\boldsymbol{z} = \boldsymbol{i}h^{\pm} + \tilde{x}$ with $\tilde{x} \in (0, \tilde{L})$ as $\partial_y\varphi = -V_0\alpha^{\pm}$. The wall lies on the x-axis. As for the body, also on the wall the condition of vanishing normal velocity is enforced. In this case, the boundary conditions are summarized as:

$$\begin{cases} 0 < \tilde{x} < \tilde{L}, \quad y = h^+ : \partial_y\varphi = -V_0\alpha^+, \\ \quad y = h^- : \partial_y\varphi = -V_0\alpha^- \\ -\infty < \tilde{x} < +\infty, \quad y = 0 : \partial_y\varphi = 0. \end{cases} \qquad (3)$$

A suitable condition at infinity has to be added to the above boundary conditions, in order to guarantee the uniqueness of the solution. This will be discussed in Section 4.

3. Analytical solution for a plate moving in compressible fluid near a free surface

In the present section, an approximate analytical solution for the flow generated by a flat plate ($\alpha^+ = \alpha^- = \alpha$) moving near a free surface is derived. The flow develops in the upper complex half-plane, the position being written as $\boldsymbol{z} := \tilde{x} + \boldsymbol{i}y$ with $y \geq 0$. The free surface lies on the real axis, the body along the segment $\boldsymbol{z} = \tilde{x} + \boldsymbol{i}h$, with $0 \leq \tilde{x} \leq \tilde{L}$, and the cavity on the same half-line for $0 \leq \tilde{x} \leq \tilde{S}$, as shown in Fig. 1. The local slope angles of the upper and lower faces of the body are defined as $\alpha^{\pm} := dh^{\pm}/dx$. In the discussion below, the body will be a flat plate, so that the upper and lower local slope angles coincide with the incidence of the plate: $\alpha^{\pm} =: \alpha$. The complex disturbance potential $\boldsymbol{\varphi}(\boldsymbol{z})$ and conjugate velocity $\overline{\boldsymbol{u}} = d\boldsymbol{\varphi}/d\boldsymbol{z}$ will be used.

The upper half-plane $\mathrm{Im}(\boldsymbol{z}) > 0$ with a cut along the half-line starting from the point $\boldsymbol{i}h$ and parallel to the positive real semi-axis is obtained by conformal mapping the upper half-plane $\mathrm{Im}(\boldsymbol{w}) > 0$ [13] with the transformation:

$$\boldsymbol{z}(\boldsymbol{w}) = \tilde{x}(\boldsymbol{w}) + \boldsymbol{i}y(\boldsymbol{w}) = \frac{h}{\pi}(\pi\boldsymbol{i} + \boldsymbol{w} - \log\boldsymbol{w} - 1). \qquad (4)$$

The negative real axis $\boldsymbol{w} = \sigma + \boldsymbol{i}0^+$ with $\sigma < 0$ goes on the real axis $\boldsymbol{z} = \tilde{x} + \boldsymbol{i}0^+$, as shown in Fig. 2 (green and blue lines). Moreover,

[1] In the following, the division by δ of any length along the x-axis will be indicated with a tilde (). Nondimensional quantities will be indicated with a star (*). The reference length is the chord of the profile (L) and the reference time is L/V_0, while for historical reasons the pressures are nondimensionalized by the dynamic pressure $\rho_0 V_0^2/2$ (ρ_0 being the fluid density far from the body).