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Influence of three-dimensional wall roughness on the transition of a finite Stokes layer[☆]

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ABSTRACT

A finite Stokes layer with a large half-width over three-dimensional wall roughness is numerically simulated by solving the perturbation equations. Simulations are conducted for different parameters that affect the shape of the wall roughness, e.g., streamwise wave number, spanwise wave number and the amplitude of the roughness. The calculations are performed at a Reynolds number of $R_\delta = 600$, which represents the transitional Reynolds number observed in experimental cases. The onset of the transition is shown to be highly correlated with the amplitude of pre-existing two-dimensional waves. It is found that the critical amplitude of the two-dimensional wave is approximately 1% of the velocity amplitude of the external flows. Moreover, the results indicate that the appearance of the transition is also affected by the fundamental wave numbers.

1. Introduction

The oscillatory boundary layer, which naturally appears in the interface between a rigid wall and fluid, e.g. the undersea and the coast, is one of the time-periodic flows. Because the exact solution of this flow was first given by Stokes G.G. (1855) [1], it has been called the Stokes layer in most studies. Due to its considerable influence on sediment transport motion and the formation of ripples, even the characteristics of geophysical flows, the Stokes layer is widely applied in a variety of fields, ranging from offshore engineering to biomedical science. Recently, the characteristics of this flow have attracted considerable attention in the theoretical and applied fields.

Stokes layers can ideally be divided into two types. The first type is called the semi-infinite Stokes layer, which is generated in a finite flat plate oscillating in its own plane. The second type is called the finite Stokes layer, which is driven by a pressure gradient in a pair of oscillating parallel plates or oscillatory pipes. The latter is frequently considered in most experimental setups. The Reynolds number in most studies is defined as $R_\delta = U_0\delta/\nu$, where U_0 is the velocity amplitude of the oscillating flat plate(s) or the external flows, and the thickness $\delta = \sqrt{2\nu/\Omega}$, where ν is the kinematic viscosity of the fluid and Ω is the angular frequency of the oscillations. For the finite Stokes layer, there is an additional parameter $h = h^*/\delta$, where h^* represents the half-width of the channels or the radius of circular pipes. Experimental observations [2–5] have shown that as the Reynolds number R_δ increases, four distinct regimes exist: (I) the laminar regime, which exists for low Reynolds numbers; (II) a disturbed laminar regime, where a small amplitude disturbance is superimposed over the laminar solution; (III) the intermittent turbulence regime, where turbulence bursting appears during the decelerating phases of the cycle; and (IV) the fully developed turbulence regime, where turbulence presents throughout the entire cycle. Many experimental results [2,6,7] have indicated that the transition from regime (II) to (III) occurs at approximately $R_\delta = 550$ for $h > 2$.

Over the past years, a number of works have been conducted to understand the linear instability of the Stokes layer, including instantaneous analysis [8–11] and Floquet theory [12–15]. In the instantaneous theory, the base flow is treated as being quasi-steady. The eigenvalue problem of the Orr–Summerfield (O–S) equation is solved at selected times. Unfortunately, the critical Reynolds number obtained by the instantaneous approach is far too low as O(10). In contrast, Floquet theory is much more rigorous for the Stokes

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layer due to the time-periodic nature of this flow. However, because of computer capacity limitations, the calculations were only performed for low Reynolds numbers in early years. In 2002, a breakthrough was achieved by Blennerhassett and Bassom [14], and the neutral curve of global stability was obtained. The Floquet unstable modes were found for a Reynolds number of $R_\delta > 1416$. Soon after, calculations were conducted for the finite Stokes layers [15], including the channels and circular pipes. Although the obtained critical Reynolds number was reduced to approximately $R_\delta = 1100$, it was still almost two times greater than the experimental result.

The above results clearly raise a question regarding the relationship between the instantaneous instability and Floquet theory. Luo and Wu (2010) [16] (henceforth referred to as LW10) considered the initial-value problem governing the evolution of an infinitesimal disturbance in a finite Stokes layer. It was shown that the energy of the disturbance was primarily carried by the instantaneous mode. In fact, it was generally believed that the transition is due to the local instability rather than the Floquet global instability. Wu (1992) [17] considered the interaction of a two-dimensional wave and a pair of oblique three-dimensional waves at sufficiently high Reynolds numbers, and the results suggested that the transition to turbulence is due to the explosive growth of the disturbance at some instants rather than relying on the growth of Floquet unstable modes. The studies on the secondary instability conducted by Akhavan et al. (1991) [18] showed that three-dimensional waves could grow if a large amplitude two-dimensional wave pre-exists in the flow. However, there was a lack of evidence explaining how the two-dimensional wave was generated. Moreover, the transition was shown to be related to the receptivity mechanism. It is believed that the transition to flows of regimes (II) and (III) is very sensitive to the experimental conditions, e.g. the external forces. Wall imperfection, as one of the common external forces, has been proposed as an explanation for the experimental observations. An analysis over two-dimensional wall imperfections was conducted by Blondeaux and Vittori (1994) [19], and the results showed that an aperiodic flow in common with the turbulence observed in experimental setups can occur through the interaction among the instantaneous unstable modes and the force modes induced by infinitesimal wall imperfections. Wu (2001) [20] considered the receptivity problem of boundary layers due to the interaction between the distributed roughness and free-stream disturbances, and some good agreements with experimental results were found.

From a numerical perspective, the transition of an oscillating Stokes layer over a three-dimensional wall roughness was investigated by Vittori and Verzicco (1998) [21] through numerical simulations, and their results indicated that wall imperfections play a fundamental role in causing the growth of the two-dimensional disturbances, which in turn trigger turbulence. LW10 considered the problem of the response of the Stokes layer to two-dimensional wall roughness. It was found that an amplitude of the roughness ε^* of only 0.1 μm will trigger more than 10% velocity perturbations for $R_\delta > 600$. Because the calculations by LW10 were conducted for a linearized equation, this problem was recently studied by Kong (2014) [22] taking into account the nonlinear effect. It was found that for $R_\delta = 600$, a value of ε^* of $O(0.1\ \mu\text{m})$ would lead to a subcritical instability of the two-dimensional waves in the Stokes layer. Thomas et al. (2014) [23] considered the Stokes layer by introducing a spatially localized impulsive forcing. The results indicated that the spatiotemporal evolution of the flow exhibited a family-tree-like structure. Recently, Thomas et al. (2015) [24] studied the linear instability problem of a Stokes layer with some superimposed high-frequency components by using direct numerical simulations and Floquet analyses. It was found that a perturbation of only 1% would be able to result in a reduction of the classical critical Reynolds number of nearly 60%.

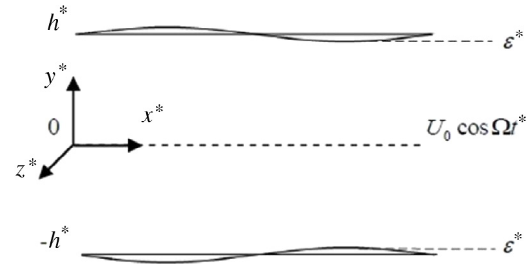

Fig. 1. Sketch of the problem.

Because the disturbances considered by Kong [22] were two-dimensional, the obtained aperiodic flow was not real turbulence. It thus raises a question regarding the three-dimensional effect. Therefore, to understand the transition, it is of great significance to study the impact of the two-dimensional disturbances with a certain level of amplitude on the three-dimensional waves. The calculations of Vittori and Verzicco [21] were performed for one parameter of the wall profile while some others were kept fixed in all runs. In the present work, three physical parameters are considered, namely, the streamwise wave number, spanwise wave number, and the height of wall roughness. The remainder of this paper is organized as follows. Section 2 presents the mathematical formulation of the problem and the numerical method; then, the verification of the numerical code will be presented in Section 3. The influence of the above three parameters on the transition is discussed in Section 3, which is divided into three subsections. Finally, in Section 4, some conclusions are given.

2. Mathematical formulation

We study a finite Stokes layer generated by a streamwise pressure gradient between two parallel flat plates with a sufficiently large distance half-width, which is shown in Fig. 1.

The streamwise pressure gradient in the channel is prescribed as

$$\frac{\partial P^*}{\partial x^*} = \rho U_0 \Omega \sin \Omega t^*, \quad \frac{\partial P^*}{\partial y^*} = 0, \quad \frac{\partial P^*}{\partial z^*} = 0 \tag{1}$$

where x^*, y^*, and z^* are the streamwise, normal and spanwise coordinates, respectively. A non-dimensional form is obtained if the coordinates (x^*, y^*, and z^*) and the half-width h^* are scaled on δ, all velocity components u, v, w on U_0, the pressure P^* on ρU_0^2, and the non-dimensional time $t = \Omega t^*$ is introduced, where ρ is the density of the fluid. If the oscillation of the flow is in the x-direction, the base flow can be written as [16]

$$U_b(y,t) = \frac{1}{2}\left(1 - \frac{\cos(1+\mathrm{i})y}{\cos(1+\mathrm{i})h}\right) e^{\mathrm{i}t} + c.c. \tag{2}$$

where h is the non-dimensional half-width of the channel and c.c. means the conjugate part. Calculations in this paper will be performed for $h = 16$, for which the instability of the finite Stokes layer represents that of the infinite one. The wall surfaces are rough, and their profiles $\xi^*(x^*, z^*)$ are given by the superimposition of sinusoidal components as follows

$$\xi^* = h^* \varepsilon (e^{\mathrm{i}\alpha x} + b e^{\mathrm{i}\beta z}) + c.c. \tag{3}$$

where α and β are the non-dimensional fundamental wave numbers in the streamwise and spanwise directions, respectively, and ε is the amplitude of the roughness, which is made dimensionless by h^*. The first cos-function of Eq. (3) corresponds to the fundamental two-dimensional disturbance, whereas the second cos-function of Eq. (3) corresponds to the fundamental