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A velocity decomposition approach for three-dimensional unsteady flow

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ABSTRACT

A velocity decomposition method is developed for the solution of three-dimensional, unsteady flows. The velocity vector is decomposed into an irrotational component (viscous-potential velocity) and a vortical component (vortical velocity). The vortical velocity is selected so that it is zero outside of the rotational region of the flow field and the flow in the irrotational region can thus be solely described by the viscous-potential velocity. The formulation is devised to employ both the velocity potential and the Navier–Stokes-based numerical methods such that the field discretization required by the Navier–Stokes solver can be reduced to only encompass the rotational region of the flow field and the number of unknowns that are to be solved by the Navier–Stokes solver is greatly reduced. A higher-order boundary-element method is used to solve for the viscous potential by applying a viscous boundary condition to the body surface. The finite-volume method is used to solve for the total velocity on a reduced domain, using the viscous-potential velocity as the boundary condition on the extent of the domain. The two solution procedures are tightly coupled in time. The viscous-potential velocity and the total velocity are time dependent due to the unsteadiness in the boundary layer and the wake. The solver is applied to solve three-dimensional, laminar and turbulent unsteady flows. For turbulent flows, the solver is applied for both Unsteady-Reynolds-Averaging-Navier–Stokes and Large-Eddy-Simulation computations.

1. Introduction

The decomposition of the velocity vector has been long exploited for the analysis of fluid flows. For example the velocity of the fluid can be decomposed into components that respectively depends on the expansion rate, vorticity, and boundary conditions of the flow field [1,2]. In [3], a velocity decomposition is applied for separation of wave and viscous resistance of ships. In [4], viscous flows are calculated by a Boundary-Element Method (BEM) using a special form of velocity decomposition so that the BEM can retain its efficiency. In [5], the decomposition of the velocity vector is used to theoretically study the evolution of water waves under the influence of viscosity. Such techniques have also been used to improve the efficiency of field methods for numerically solving the Navier–Stokes equations [6–14].

In this work, the velocity decomposition algorithm first introduced in [15], is extended to solve three-dimensional (3D), unsteady flow problems. The main difference that sets our method apart from other similar methods is the ability to fully describe the flow field external to the rotational region through a viscous potential which is solved by using a BEM on the physical-body boundary. The motivation for this is that the field discretization required by the Navier–Stokes solver can be reduced to only encompass the rotational part of the flow region. Our target applications are external incompressible flows with high Reynolds number, such as those that appear in naval and automotive engineering. The portion of the flow field that is rotational only resides within a small region around the body and the wake downstream of the body, while the rest of the flow domain is irrotational and can be described by a scalar potential function. For a description of the development of our velocity decomposition approach, see [15,16].

In previous work, [15,6,16,9], the velocity decomposition method has been developed to address steady flows of two-dimensional (2D), axis-symmetric, non-lifting bodies without water waves, or 2D bodies that can have lift and be near a water surface. In this paper, the algorithm is extended to address 3D, unsteady flow problems. To solve for 3D flows, a model for 3D viscous potential is necessary. A higher-order boundary-element solver based on B-splines is used instead of the 2D panel method that was used in previous studies. The unsteadiness of the flow

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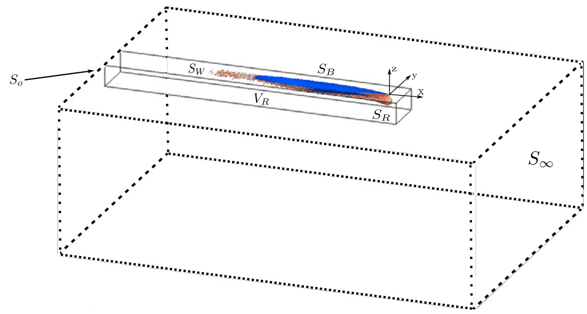

Fig. 1. The schematic of the computational domain.

field is calculated using a tightly coupled algorithm previously outlined for 2D flows in [17]. Only cases with non-accelerating bodies are presently considered in this work. The unsteadiness in the solutions are due to the fluid flow itself, such as the development of the boundary layer, separation, turbulence and vortex shedding. The method presented in this work can be directly applied to cases with bodies that are accelerating.

The paper is outlined as follows. In Section 2, the formulation of the Navier–Stokes problem of interest is described. More specifically, the velocity decomposition for 3D, unsteady flows is introduced, and the resultant viscous potential and Navier–Stokes sub-problems are stated. In Section 3, the numerical implementation of the solution procedures for the sub-problems and the coupling algorithm are presented. In Section 4, the algorithm is tested on a set of example problems. The 3D viscous potential is calculated and examined using several test cases, and the differences between our viscous potential and the inviscid potential are demonstrated. The 3D, turbulent and laminar flows over a finite flat plate are studied. Turbulent, double-body flow over a Wigley hull is calculated within the Large-Eddy-simulation frame work. For all the test cases, the velocity decomposition solver is used to calculate the flow field within a greatly reduced domain. The reduced domain results are compared with the corresponding Navier–Stokes solution calculated in sufficiently large domains using a conventional solver. Section 5 contains the summary and conclusions.

2. Problem formulation

For the problem considered in this work, viscous effects, such as the boundary layer and viscous wake, are assumed to be confined within a small region around the body and the fluid flow is irrotational for the rest of the domain. This is a suitable assumption for many high Reynolds number applications such as those with automobiles, trains, aerospace and marine vehicles, etc. The flow field of interest is governed by the incompressible Navier–Stokes equations, shown in Eqs. (1) and (2).

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u} \otimes \mathbf{u}) = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} \tag{2}$$

where $\mathbf{u} = \mathbf{u}(\mathbf{x}, t)$ is the total velocity vector field that varies in space and time, $p = p(\mathbf{x}, t)$ is the pressure, ρ is the density of the fluid and ν is the kinematic viscosity.

The schematic of the computational domain is shown in Fig. 1. A body-fixed coordinate system is used and the position vector is $\mathbf{x} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$. On the body boundary S_B, the no-slip boundary condition, Eq. (3), is enforced.

$$\mathbf{u} = 0 \quad \text{on } S_B. \tag{3}$$

The disturbance due to the presence of the body disappears far away from the body, so the far-field boundary condition, Eq. (4), is applied on S_∞.

$$\lim_{|x|\to\infty} \mathbf{u} = U_\infty\,\hat{\mathbf{i}}. \tag{4}$$

The initial condition is prescribed through an initial velocity field $\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0(\mathbf{x})$, where $\mathbf{u}_0$ is the initial velocity field.

Throughout this paper, the problem described above is designated as the *Navier–Stokes problem*, which consists of the equations, domain and the boundary conditions. The *Navier–Stokes problem* also refers to situations in which a varied form of the governing equations are used such as when the turbulence is modeled through either the Unsteady-Reynolds-Averaged-Navier–Stokes (URANS) equations and Large-Eddy-Simulation (LES) Navier–Stokes equations.

The velocity decomposition used in this work is similar to the one used in [9], except that all three velocities are time dependent as shown in Eq. (5), where $\nabla\varphi$ and $\mathbf{w}$ are the viscous-potential velocity and the vortical velocity respectively. All three velocities are assumed to vary in time but not necessarily with the same time scale. The time dependence of each term in Eq. (5) is discussed in more detail in Section 3.3. All three components of the velocity are divergence free.

$$\mathbf{u}(\mathbf{x}, t) = \nabla\varphi(\mathbf{x}, t) + \mathbf{w}(\mathbf{x}, t). \tag{5}$$

The velocity decomposition is not uniquely defined without proper boundary conditions [18]. Various forms can be devised for different calculation purposes [4]. In our work we seek a decomposition such that the vortical velocity $\mathbf{w}$ is zero outside the rotational region of the flow field, Eq. (6).

$$\mathbf{w} = 0 \quad \text{for } |\mathbf{x}| \geq \delta. \tag{6}$$

In viscous flows, the vorticity ω vanishes exponentially at infinity [1]. Hence the irrotational region defined in this work is where the vorticity is negligible for all practical purposes, [19]. Then the vortical region denotes the complement of the irrotational region. The vorticity thickness, (i.e. the body-normal distance away from the body boundary where the flow becomes practically irrotational) is denoted as δ. The vortical velocity $\mathbf{w}$ can be uniquely defined by applying the boundary condition Eq. (7) on the boundary of the vortical region, S_δ. (The illustration of δ and S_δ can be found in Fig. 2.) Then the decomposition is also uniquely defined for a given velocity field and the viscous-potential velocity $\nabla\varphi$ can fully describe the flow field outside the vortical region Eq. (8).

$$\mathbf{w} \cdot \hat{n} = w_n = 0 \quad \text{on } S_\delta \tag{7}$$

$$\nabla\varphi = \mathbf{u} \quad \text{for } |\mathbf{x}| \geq \delta. \tag{8}$$

The Navier–Stokes problem is decomposed into a Navier–Stokes sub-problem and a viscous potential sub-problem by applying the decomposition stated in Eq. (5). The Navier–Stokes sub-problem is defined similarly as the Navier–Stokes problem, except that it is defined within a greatly reduced domain, V_R. The boundary of the reduced domain, S_R, is chosen to encompass the rotational region of the flow field. The velocity boundary condition Eq. (9) on the reduced domain boundary is prescribed through the viscous-potential velocity.

$$\mathbf{u}(\mathbf{x}, t) = \nabla\varphi(\mathbf{x}, t) \quad \text{on } S_R. \tag{9}$$

The total velocity vector $\mathbf{u}$ is calculated by solving the Navier–Stokes sub-problem together with a BVP (Boundary-Value Problem) that governs φ.

The unknown variable we want to solve in the viscous potential sub-problem is the viscous potential φ. The governing equation is