European Journal of Mechanics B/Fluids 62 (2017) 139–148

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Optimal wavy surface to suppress vortex shedding using second-order sensitivity to shape changes

O. Tammisola *

KTH Mechanics, KTH Royal Institute of Technology, SE-10044 Stockholm, Sweden
Faculty of Engineering, The University of Nottingham, University Park, Nottingham NG7 2RD, UK

ARTICLE INFO

Article history:
Received 7 December 2015
Received in revised form
10 July 2016
Accepted 15 December 2016
Available online 23 December 2016

Keywords:
Flow instability
Flow control
Sensitivity
Symmetry-breaking

ABSTRACT

A method to find optimal 2nd-order perturbations is presented, and applied to find the optimal spanwise-wavy surface for the suppression of cylinder wake instability. As shown in recent studies (Hwang et al., 2013, Tammisola et al., 2014, Del Guercio et al., 2014), 2nd-order perturbations are required to capture the stabilizing effect of spanwise waviness, which is ignored by standard adjoint-based sensitivity analyses. Here, previous methods are extended so that (i) 2nd-order sensitivity is formulated for base flow changes satisfying the linearized Navier–Stokes, and (ii) the resulting method is applicable to a 2D global instability problem. This makes it possible to formulate the 2nd-order sensitivity to shape modifications. This formulation is used to find the optimal shape to suppress the cylinder wake instability. The optimal shape is then perturbed by random distributions in full 3D stability analysis to confirm that it is a local optimal at the given amplitude and wavelength. At $Re = 100$, surface waviness of maximum height 1% of the cylinder diameter is sufficient to stabilize the flow. The optimal surface creates streaks passively by extracting energy from the base flow derivatives and altering the tangential velocity component at the wall. This paper extends previous techniques to a fully two-dimensional method to find boundary perturbations which optimize the 2nd-order drift. The method should be applicable to generic flow instability problems, and to different types of control, such as boundary forcing, shape modulation or suction.

1. Introduction

The present study deals with optimal second-order eigenvalue drifts, which may arise as a result of asymmetric control. One example is the flow around a cylinder with spanwise-sinusoidal boundary modulations—the demonstration case in the present study. Since a decade, spanwise waviness is known to efficiently suppress vortex shedding and reduce drag behind bluff bodies. [1] showed experimentally that a spanwise wavy trailing edge completely suppressed the vortex shedding around a rectangular cylinder at $Re = 40\,000$, resulting in a 30% reduction of the mean drag. A similar effect was observed by [2] numerically at $Re = 100$–500.

As pointed out by [2], the stabilizing effect of spanwise waviness may also be created by changing the wall boundary condition by bleed or transpiration. Through steady spanwise-alternating suction and blowing, [3] shifted the Hopf bifurcation of the wake behind a circular cylinder from $Re \approx 45$ to $Re > 140$ in DNS. The instability could only be suppressed when the actuation had a spanwise wavelength of 5–6 cylinder diameters. The reason for the efficiency of medium wavelengths has been analysed in several subsequent works. [4] examined the instability of a fixed wake profile superposed with spanwise waviness, and observed that in this model medium wavelengths were not absolutely unstable. [5,6] considered base flow modifications generated by spanwise-alternating suction. They concluded that the streaks generated by suction were optimally amplified by transient growth at medium wavelengths, and hence the base flow modification was also largest at medium wavelengths. [7] considered formally modifications of global mode eigenvalues with spanwise-wavy base flow modifications, which required 2nd-order perturbations. Wavelength selection was based on an *eigenmode resonance* at long wavelengths, and the strongest interaction with the 2nd-order sensitivity core at medium wavelengths.

The optimal distribution of spanwise waviness has been studied much less than the optimal wavelength. However, for the flow around the circular cylinder, azimuthal location of the waviness is an important parameter. [3] applied the spanwise-alternating suction from two slots placed on the top and the bottom of the cylinder; locations at the rear and front of the cylinder were

* Correspondence to: KTH Mechanics, KTH Royal Institute of Technology, SE-10044 Stockholm, Sweden.
E-mail address: outi@mech.kth.se.

http://dx.doi.org/10.1016/j.euromechflu.2016.12.006
0997-7546/

mentioned to be inefficient. Moreover, the configuration in which the suction through the upper slot was in-phase with that through the lower slot was found to be much more effective than the anti-phase configuration, which was later explained using the mode resonance effect in [7]. [6] performed a 3D optimization of the azimuthal distribution of waviness in order to create strongest possible base flow streaks. Their optimal distribution also peaked at the top and bottom of the cylinder but was continuous, and stabilized the flow at a much lower peak suction amplitude (<1%) than the slots of [3] (8%) at $Re = 100$. However, the optimization was performed on the streakiness of the base flow, and eigenvalue drift was not a part of the optimization. Hence, the method was dependent on the physics of this particular flow and the chosen control.

The aim of the present study is to extend the previous works into a method of mathematically optimizing the eigenvalue drift, so that the method would be applicable for other flow cases and other choices of (asymmetric) boundary control. [8] computed optimal spanwise-wavy base flow modifications for a parallel flow in a mixing layer, accounting for the eigenvalue drift. The 2nd-order perturbation system was written in matrix form and elegantly manipulated to form a Hessian matrix, and the most stabilizing perturbation found from its extremal eigenpairs. The manipulations involved forming an explicit inverse of a system matrix, which was possible since the flow was parallel with 1D eigenfunctions. The global wake instability problem considered here, however, has 2D eigenfunctions.

The present study introduces a new approach to compute optimal boundary perturbations at the 2nd order, accounting for both base flow change and eigenvalue drift. The perturbation system is projected on a smaller basis of boundary functions, and the optimal recovered using only 2D computations no larger than the original system. Using this method, we find the optimal spanwise-wavy cylinder surface to suppress vortex shedding around it. Spanwise-wavy shapes are already used to suppress vortex shedding around *e.g.* chimneys. The optimal spanwise-wavy shape, however, has not been examined yet.

The new attributes of this approach can be summarized as follows. In [7], base flow modifications induced by wall suction were computed and analysed a posteriori. In [8], 2nd order optimal base flow modifications were computed a priori. Their base flow sensitivity was the 2nd-order counterpart of generic base flow sensitivity [9], in particular, the base flow modifications did not satisfy Navier–Stokes equations. The present theory introduces base flow modifications which satisfy the (boundary-perturbed) Navier–Stokes equations, coupling this with the maximal eigenvalue drift, similarly to an adjoint base flow approach [9]. In addition, the projection to boundary basis functions makes it possible to apply the optimization to two-dimensional problems. An extension to optimal shapes containing both 1st-order (spanwise constant) and 2nd-order (spanwise wavy) shape modifications is relatively straightforward.

2. Perturbation analysis

Let us consider a general eigensystem of the form:

$$\mathcal{L}^{(0)}\{\mathbf{q}^{(0)}\} = \sigma^{(0)}\mathbf{q}^{(0)}, \tag{1}$$

where $\mathbf{q}^{(0)}$ is an eigenvector, and $\sigma^{(0)}$ an eigenvalue, and the curly bracket indicates that the operator $\mathcal{L}^{(0)}$ acts on $\mathbf{q}^{(0)}$—this notation is adopted throughout the paper. After introducing a small boundary modification denoted by ϵh, where ϵ is an amplitude parameter and h is normalized to unity in a given boundary norm, we write:

$$\mathcal{L}(\epsilon h)\,\{\mathbf{q}(h)\} = \sigma(h)\mathbf{q}(h). \tag{2}$$

The solution may be expanded in a perturbation series where ϵ denotes the amplitude of h (*e.g.* [10]):

$$\left(\mathcal{L}^{(0)} + \epsilon\mathcal{L}^{(1)} + \epsilon^2\mathcal{L}^{(2)} + O(\epsilon^3)\right)\left\{\sum_{n=0}^{2}\epsilon^n\mathbf{q}^{(n)} + O(\epsilon^3)\right\} = \left(\sum_{n=0}^{2}\sigma^{(n)} + O(\epsilon^3)\right)\left(\sum_{n=0}^{2}\epsilon^n\mathbf{q}^{(n)} + O(\epsilon^3)\right) \tag{3}$$

where the superscripts in parenthesis, (0), (1), (2), denote indices in the perturbation series (while powers are written without parenthesis). By grouping together terms of any given power of ϵ, we can generate approximations of the eigenvalue drift accurate up to that order.

1st *order*: When collecting terms of the order ϵ^1, we obtain:

$$\left(\mathcal{L}^{(0)} - \sigma^{(0)}\mathcal{I}\right)\{\mathbf{q}^{(1)}\} = -\mathcal{L}^{(1)}\{\mathbf{q}^{(0)}\} + \sigma^{(1)}\mathbf{q}^{(0)}, \tag{4}$$

where $\mathcal{I}$ is the identity operator. By projecting this equation under inner product $\langle , \rangle$ with the adjoint eigenmode $\mathbf{q}^{(0)+}$, it can be shown [10] that the left hand side is zero: $\langle \mathbf{q}^{(0)+}, \left(\mathcal{L}^{(0)} - \sigma^{(0)}\mathcal{I}\right)\{\mathbf{v}\}\rangle = 0$ for any vector $\mathbf{v}$. By equating the right-hand side with zero, the 1st-order eigenvalue drift $\sigma^{(1)}$ is found to be:

$$\sigma^{(1)} = \langle \mathbf{q}^{(0)+}, \mathcal{L}^{(1)}\{\mathbf{q}^{(0)}\}\rangle, \tag{5}$$

which is equivalent to the integrated *sensitivity* used to estimate an eigenvalue drift with respect to control in numerous previous studies (see *e.g.* [11] for a review).

2nd *order*: When collecting terms of the order ϵ^2, we obtain:

$$\left(\mathcal{L}^{(0)} - \sigma^{(0)}\mathcal{I}\right)\{\mathbf{q}^{(2)}\} = -\mathcal{L}^{(1)}\{\mathbf{q}^{(1)}\} - \mathcal{L}^{(2)}\{\mathbf{q}^{(0)}\} + \sigma^{(1)}\mathbf{q}^{(1)} + \sigma^{(2)}\mathbf{q}^{(0)}. \tag{6}$$

Again, the left hand side is zero when projected by $\mathbf{q}^{(0)+}$. By equating the right hand side to zero, $\sigma^{(2)}$ takes the form:

$$\sigma^{(2)} = \langle \mathbf{q}^{(0)+}, \mathcal{L}^{(1)}\{\mathbf{q}^{(1)}\}\rangle + \langle \mathbf{q}^{(0)+}, \mathcal{L}^{(2)}\{\mathbf{q}^{(0)}\}\rangle, \tag{7}$$

where $\mathbf{q}^{(1)}$ is the 1st-order eigenvector correction obtained from Eq. (4), normalized so that $\langle \mathbf{q}^{(0)+}, \mathbf{q}^{(1)}\rangle = 0$.[1] The second term represents a *change* in the 1st-order eigenvalue drift, in the case that $\mathcal{L}$ depends quadratically on the boundary modification. To find $\mathcal{L}^{(1)}$ and $\mathcal{L}^{(2)}$, the governing linear operator $\mathcal{L}(h)$ needs to be Taylor-expanded so that:

$$\mathcal{L} = \mathcal{L}^{(0)} + \epsilon\mathcal{L}^{(1)} + \epsilon^2\mathcal{L}^{(2)} + O(\epsilon^3) \tag{8}$$

where $\mathcal{L}^{(1)}$ is linear in h, and $\mathcal{L}^{(2)}$ is a symmetric bilinear form in h, in the boundary inner product to be defined soon.

Shape changes, boundary suction, or mass injection at the cylinder can all be addressed by the method presented next with minimal adjustments to the boundary conditions. It is common in shape optimization to parameterize the boundary, to reduce the degrees of freedom, but also obtain robust optimal shapes which are easy to manufacture [12]. Let us parameterize the displacement of the cylinder wall using N basis functions:

$$\epsilon h = \sum_{n=1}^{N} a_n h_n. \tag{9}$$

[1] Which implies that $\sigma_1\langle \mathbf{q}_0^+, \mathbf{q}_1\rangle = 0$.