Comput. Methods Appl. Mech. Engrg. 198 (2008) 2–13

Contents lists available at ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Computational methods in optimization considering uncertainties – An overview

G.I. Schuëller [a,*], H.A. Jensen [b]

[a] Chair of Engineering Mechanics, University of Innsbruck, Technikerstr. 13, A-6020 Innsbruck, Austria
[b] Department of Civil Engineering, Santa Maria University, Casilla 110-V, Valparaiso, Chile

A R T I C L E I N F O

Article history:
Received 10 April 2008
Accepted 6 May 2008
Available online 15 May 2008

Keywords:
Uncertainty
Optimization
Robustness
Reliability-based optimization
Robust design optimization
Model updating

A B S T R A C T

This article presents a brief survey on some of the most relevant developments in the field of optimization under uncertainty. In particular, the scope and the relevance of the papers included in this Special Issue are analyzed. The importance of uncertainty quantification and optimization techniques for producing improved models and designs is thoroughly discussed. The focus of the discussion is in three specific research areas, namely reliability-based optimization, robust design optimization and model updating. The arguments presented indicate that optimization under uncertainty should become customary in engineering design in the foreseeable future. Computational aspects play a key role in analyzing and modeling realistic systems and structures.

1. Introduction

In most engineering applications, the traditional approach to designing systems is to consider deterministic models and parameters, respectively. Thus, variations in loading conditions, material properties, geometry, boundary conditions, etc. are included in the design process by introducing simplifying hypotheses, e.g. the consideration of extreme or mean values and/or the application of safety factors. These hypotheses are formulated based on past experience and engineering judgement. Despite such a traditional approach has successfully been used in many practical design situations, the assumption of a deterministic model is certainly a simplification, because observations and measurements of physical processes clearly show variability and randomness in the different model parameters. Hence, a proper design procedure must explicitly consider these types of uncertainties, as they may cause significant changes in the performance and reliability of final designs (see, e.g. [31,100,113,133]). For example, final designs obtained by deterministic models may become infeasible when the uncertainty in the system parameters is considered. The application of design procedures which consider uncertainties ensures that the analyzed system will perform within prescribed margins with a certain reliability, i.e. a quantitative measure of the system safety will be available.

Despite of the fact that an adequate level of reliability is a basic objective when designing a system, other design goals may be important as well, e.g. there is an increasing demand for structures which are safer and at the same time more economical. In consequence, engineering practice expects to have optimization procedures available which take into account the effects of uncertainty and which are applicable to realistic problems of the engineering practice.

Procedures which deal with optimization considering uncertainties are significantly more involved than their deterministic counterparts. Optimization processes may require the evaluation of costly objective and constraint functions hundreds or even thousands of times. The associated costs are usually prohibitive, especially under uncertain conditions, e.g. when the system is represented by means of a large and detailed finite element model or when the representation of the loading acting on a structure requires a numerically involved model, such as in earthquake engineering applications. Therefore, special procedures must be applied in order to make the design problem tractable. Such procedures include, for example:

- The use of efficient optimization techniques which require less function calls. These techniques can take advantage of special characteristics of the problem under study by introducing, e.g. sequential approximations, construction of approximate representations of the objective function and constraints using reciprocal and/or hybrid variables, etc.
- The introduction of approximation concepts at different levels of the optimization process.
- The use of appropriate techniques for coping with uncertainty, e.g. simulation techniques that allow to treat realistic uncertainty models involving a large number of uncertain parameters in an efficient manner.

* Corresponding author. Tel.: +43 512 507 6841; fax: +43 512 507 2905.
E-mail address: Mechanik@uibk.ac.at (G.I. Schuëller).

0045-7825/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.cma.2008.05.004

- An appropriate computational implementation, i.e. computational aspects play a key role, as the systems and structures which are of engineering interest are large and require detailed modeling. In this regard, parallel computing has become a tool which is steadily gaining interest among researchers and engineers.

Procedures are developed to a point where their application to realistic problems is now feasible. This certainly suggests a change of paradigm when performing optimization under uncertainty, as early approaches were restricted to academic examples. On the contrary, the most recent methods developed are being applied to challenging engineering problems. Hence, it is foreseen that optimization would become an integral part in the field of computational stochastic mechanics, allowing to treat realistic design applications within an appropriate and efficient framework.

This article is not intended to present an exhaustive review of the field of optimization under uncertainties but to offer a brief survey on some of the most relevant contributions in the area, namely reliability-based optimization (RBO), robust design optimization (RDO) and model updating. Thus, topics such as non-probabilistic approaches (e.g. Fuzzy analysis) for coping with uncertainty were not considered, as the focus of this Special Issue is on classical probability analysis and Bayesian approaches. In the same way, fields such as life-cycle optimization of structures were not further pursued, as it was intended to show recent advances on basic methods.

As previously pointed out, this volume includes three areas of research in the field of optimization under uncertainties. The first area, *reliability-based optimization*, is concerned with the solution of an optimization problem, where the effects of uncertainties are quantitatively expressed by means of failure probabilities and expected values [36,42,58,117]. The second area addressed in this volume refers to *robust design optimization*, which is a methodology that seeks to determine a design which is relatively insensitive with respect to changes in the loading, structural parameters, geometry, etc.; such a design is referred to as a *robust* solution [33]. The third and final area covered in this survey refers to *model updating* and *system identification*. In this research field, the goal is to reduce the discrepancies that arise when comparing the model predictions with test data [38,39,51].

2. Reliability-based optimization

2.1. General remarks

The basic goal in any engineering discipline is to design and construct systems or components that satisfy certain performance objectives during their lifetime. Such objectives cover a wide range of possibilities, e.g. control of vibrations induced by wind or traffic loading on bridges, collapse prevention of buildings due to major earthquakes, minimization of the effects of multi-site damage in aerospace structures, etc. In almost any practical design situation it is impossible to comply with the performance objectives deterministically because of the inherent random nature of loading conditions, structural parameters and/or conditions of operation of the structures. Hence, the fulfillment of the performance objectives can be accomplished only by probabilistic means, i.e. with an associated reliability. In fact, high levels of reliability are usually associated with large economical costs, e.g. a structure with an enhanced reliability may require the use of an increased amount of construction material, more sophisticated construction procedures, thorough maintenance, etc. Considering that the available resources are always scarce, an adequate design procedure should offer an appropriate trade-off between an acceptable reliability level and economical design of the structure. Reliability-based optimization (RBO) provides the means for achieving such trade-off by offering an optimal design solution taking into account the effects of uncertainties [36,42,58,117,144,153,157].

The RBO approach is an attractive and most useful design tool: it allows to determine the *best* design according to some predefined criterion. The formulation of an RBO problem requires the identification and definition of a number of items, namely the input variables of the system (i.e. *design* variables and *uncertain* parameters), the failure events of the system (i.e. violation of target performance), the constraints of the design problem and the objective function that allows to identify the most convenient design. Each of these items is briefly described below.

- *Definition of the design variables* ($\boldsymbol{y}$). The design variables are those parameters that can be selected by the designer and that affect the performance of a system. Typical examples of design variables are the cross sections of structural members, interval of inspection and repair, topology parameters, etc. The design variables can be characterized as deterministic or uncertain; in the latter case, the mean value is usually set as the design variable.
- *Identification of the uncertain parameters*. In any practical situation there are a number of parameters which are not known at the design stage and that affect the performance of a system. Such parameters may refer to loadings (e.g. wind loading, seismic events, water wave loading, etc.), structural parameters (e.g. geometry of a system, yield strength, etc.), and operation conditions (e.g. temperature, environmental conditions, etc.) among others. These parameters are characterized as uncertain variables ($\boldsymbol{\theta}$). The rational quantification of the effects of these parameters in the system performance requires an appropriate model to measure the plausibility of a given realization of $\boldsymbol{\theta}$, e.g. by means of the so-called non-probabilistic approach (see, e.g. [103]) or by prescribing a joint probability density function $f(\boldsymbol{\theta})$.
- *Formulation of the failure (or critical) events associated with the performance of the system*. As mentioned above, the system should fulfill certain performance requirements. The violation of any of these requirements causes a failure of the system. It should be noted that, in this context, failure does not necessarily imply collapse but rather an undesirable performance. A failure event is defined by means of the so-called performance function g, which depends on the design variables and the uncertain parameters, i.e. $g = g(\boldsymbol{y}, \boldsymbol{\theta})$. The performance function is defined such that $g(\boldsymbol{y}, \boldsymbol{\theta}^*)$ is smaller or equal to zero when a specific realization of $\boldsymbol{\theta}^*$, in combination with a specific set of design variables $\boldsymbol{y}$, causes failure (i.e. an unacceptable performance of the system); in case of an acceptable performance, $g(\boldsymbol{y}, \boldsymbol{\theta}^*)$ is larger than zero. It is important to note that a realistic reliability model of a system may involve the definition of several failure events.
- *Definition of the constraints of the design problem*. The objective of the constraints is to restrict or confine the design variable space in order to attain certain specific design requirements. The constraints can be of deterministic nature when they refer exclusively to the design variables. Deterministic constraints are formulated as a mathematical function ($h(\boldsymbol{y})$); usually, a deterministic constraint is defined such that $h(\boldsymbol{y}) \leqslant 0$ implies the satisfaction of a constraint.The constraints may also include both design variables and uncertain parameters. In such cases, the constraint is probabilistic and its fulfillment will be associated with one (or more) of the failure events defined above. Thus, a probabilistic constraint is satisfied when the probability of occurrence of the failure event ($p_F(\boldsymbol{y})$) is equal or smaller than a prescribed probability level (p_F^*), i.e. the constraint is satisfied