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ScienceDirect
Comput. Methods Appl. Mech. Engrg. 198 (2008) 52–71
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Approximate optimization of systems with high-dimensional uncertainties and multiple reliability constraints

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Received 14 August 2007; received in revised form 14 January 2008; accepted 16 January 2008
Available online 26 January 2008

Abstract

A novel approach is proposed to solve reliability-based optimization (RBO) problems where the uncertainty dimension can be large and where there may be many reliability constraints. The basic idea is to transform all reliability constraints in the target RBO problem into non-probabilistic ordinary ones by a pilot analysis. It will be shown that such a pilot analysis only requires a single run of the modified subset simulation (called the parallel subset simulation) regardless the number of the reliability constraints. Once the reliability constraints are approximated by the ordinary ones, the RBO problem can be solved as if it is an ordinary optimization problem. The resulting optimal solution should be approximately feasible, and the corresponding objective function value is minimized under the approximate constraints. Three numerical examples are investigated to verify the proposed novel approach. The results show that the approach may be capable of finding approximate solutions that are usually close to the actual solution of the target RBO problem.
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Keywords: Reliability-based optimization; Subset simulation; Stochastic simulation; High dimensions

1. Introduction

Uncertainties are abundant in civil engineering systems. Reliability-based optimization (RBO) [1–5] has become an important research area recently because of the need of making decisions under these uncertainties. For performance-based engineering [6,7], the performance measures are often uncertain, so any decision making based on them may involve RBO. In life-cycle engineering [8–11], the decision of maintenance schedule is often a RBO problem. Reliability-based optimal control [12,13] is another example of RBO.

Although RBO is the key to many research areas, its development is not yet mature for real applications due to the following issues: (a) the main difficulty encountered for RBO is in the reliability constraints, to directly ensure which during the optimization algorithm may require many reliability analyses executed at various design values. The required computational cost of doing so can be huge, rendering many realistic RBO problems computationally intractable; (b) another difficulty lies in the fact that reliability methods based on design points, e.g. the first-order and second-order reliability methods, are inefficient for problems with high-dimensional uncertainties and multiple design points, so RBO methods based on them are inefficient to practical problems with many uncertainties; (c) furthermore, the recent trend of performance-based engineering makes RBO problems even more challenging: there are usually multiple performance requirements, so the resulting RBO problems may have multiple reliability constraints. Unfortunately, most available RBO methods cannot efficiently handle multiple reliability constraints. These difficulties may explain why many design examples in RBO literature are academic type, i.e. simple systems subject to few uncertainties and a single reliability constraint.

A solution to the difficulty (a) is to convert the reliability constraint into an ordinary constraint by first estimating the entire function that relates failure probabilities with design

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doi:10.1016/j.cma.2008.01.004

variables. This approach was taken in Jensen [5], where the logarithm of the failure probability function is assumed to be linear in the design variables θ for a deterministic linear system subject to stochastic excitation. The number of the to-be-determined coefficients in the linear function is equal to the dimension of θ, denoted by n_θ, so at least n_θ reliability analyses are required to determine those coefficients. In Gasser and Schueller [2], a quadratic function is assumed, so the minimum required number of reliability analyses to determine the coefficients rapidly increases to $n_\theta + n_\theta(n_\theta + 1)/2$. The similar approach was also taken with the response surface methods or surrogate-based methods [14,15]. The required number of reliability analyses can be also large. However, large number of reliability analyses can be computationally costly. Ching and Hsieh [16] propose a novel way, where the logarithm of the failure probability function is assumed to be linear, of locally estimating failure probability function with a single simulation-based reliability analysis. This method greatly reduces the number of reliability analyses. The afore-mentioned methods share the same feature: they are local methods because the logarithmically linear or quadratic assumption may be proper only locally, not globally, so they make take several trials to find the approximate optimal solution. The difficult (b) can be easily overcome if a simulation-based RBO method is taken. However, the difficulty (c) is quite challenging, and to the authors' knowledge, there is no efficient solution available at this time.

In this paper, we propose a novel approach of solving practical RBO problems for which the associated uncertainties can be high-dimensional, the target system can be arbitrarily complex, e.g. static and dynamical highly nonlinear systems, and there may be many reliability constraints. Surprisingly, it is shown that in order to obtain an approximate solution of such a challenging RBO problem, a single run of a modified version of subset simulation (SubSim) [17–19] suffices. The main idea is to use the single pilot run of the modified SubSim to approximate all reliability constraints with ordinary nonlinear constraints by a theorem of equivalence between reliability and safety factor. Once all constraints are turned into ordinary ones, the RBO problem can be solved approximately using a suitable deterministic optimization algorithm to find the approximate solution of the target RBO problem.

2. Definition of multi-constraint RBO problems

Let Z be the uncertain variables of the target system and θ be the design parameters. Given the design parameters θ, the probability of failure of the target system is

$$P_F(\theta) = P(F|\theta) = \int_{\Omega_F} p(Z|\theta)\mathrm{d}Z, \tag{1}$$

where F denotes the failure event: $F \equiv \{R[Z,\theta] > 1\}$; $R[Z,\theta]$ is called the limit-state function; Ω_F is the failure domain in the Z space. The limit-state function does not necessarily define the complete collapse of the system but the performance of the system, e.g. serviceability or ultimate capacity. Throughout the paper, it is assumed without loss of generality that $R[Z,\theta]$ is positive and that the probability density function (PDF) of the uncertainty Z conditioning on θ, denoted by $p(Z|\theta)$, is known. A reliability-based optimization problem with multiple reliability constraints is to solve the following:

$$\begin{aligned} \min_{\theta} \quad & c_0(\theta) \\ \text{s.t.} \quad & P_{F,j}(\theta) \leqslant P^*_{F,j} \quad j = 1,\ldots,M, \\ & c_l(\theta) \leqslant 0 \quad l = 1,\ldots,L, \end{aligned} \tag{2}$$

where $c_0(\theta)$ is the objective function; $\{c_l(\theta) \leqslant 0 : l = 1,\ldots,L\}$ are deterministic constraints of θ; $\{P_{F,j}(\theta) \leqslant P^*_{F,j} : j = 1,\ldots,M\}$ are the reliability constraints, where $P^*_{F,j}$ is the target probability of failure for the j-th reliability constraint.

The RBO problem in (2) cannot be solved using common optimization algorithms because of the reliability constraints. An obvious way of solving the RBO problem is to conduct a search, which may or may not require evaluating the gradients and Hessians of the functions in (2), in the θ space. This approach was adopted by Papadrakakis and Lagaros [3], Tsompanakis and papadrakakis [20], Youn et al. [21], etc. The drawback of this approach is that it may require numerous reliability analyses to ensure the reliability constraints are satisfied during the search process.

On the other hand, if the failure probability functions $\{P_{F,j}(\theta) : j = 1,\ldots,M\}$ can be estimated beforehand, the reliability constraints can then be transformed into ordinary constraints, so the RBO problem can be turned into an ordinary optimization problem that can be solved using common optimization algorithms. This approach was taken by Gasser and Schueller [2] and Jensen [5], where the failure probability functions are estimated beforehand using many reliability analyses. In Jensen [5], the logarithm of the failure probability function is assumed to be linear, so the number of the to-be-determined coefficients is equal to n_θ for each reliability constraint, and at least n_θ reliability analyses are required to determine those coefficients for each reliability constraint. In Gasser and Schueller [2], a quadratic function is assumed, so the minimum required number of reliability analyses rapidly increases to $n_\theta + n_\theta(n_\theta + 1)/2$ for each reliability constraint. The similar approach was also taken with the response surface methods or surrogate-based methods [14,15]. The required number of reliability analyses can be also large.

In this paper, the proposed novel approach is able to transform all reliability constraints into ordinary ones with a single pilot run of SubSim regardless of the number of reliability constraints. The transformation uses a theorem of equivalence between reliability and factor of safety. In the next section, this theorem is presented.

3. Equivalence between reliability and factor of safety

Let D be the prescribed allowable design region in the θ space. Let us further define the nominal limit-state function

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