Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 198 (2008) 72–87
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

An efficient reliability-based optimization scheme for uncertain linear systems subject to general Gaussian excitation

H.A. Jensen [a,*], M.A. Valdebenito [b], G.I. Schuëller [b]

[a] *Department of Civil Engineering, Santa Maria University, Casilla 110V, Valparaiso, Chile*
[b] *Chair of Engineering Mechanics, University of Innsbruck, Technikerstrasse 13, A-6020 Innsbruck, Austria*

Received 12 September 2007; received in revised form 11 January 2008; accepted 16 January 2008
Available online 26 January 2008

Abstract

A very efficient methodology to carry out reliability-based optimization of linear systems with random structural parameters and random excitation is presented. The reliability-based optimization problem is formulated as the minimization of an objective function for a specified reliability. The probability that design conditions are satisfied within a given time interval is used as a measure of the system reliability. Approximation concepts are used to construct high quality approximations of dynamic responses in terms of the design variables and uncertain structural parameters during the design process. The approximations are combined with an efficient simulation technique to generate explicit approximations of the reliability measures with respect to the design variables. In particular, an efficient importance sampling technique is used to estimate the failure probabilities. The number of dynamic analyses as well as reliability estimations required during the optimization process are reduced dramatically. Several example problems are presented to illustrate the effectiveness and feasibility of the suggested approach.

Keywords: Approximation concepts; Gaussian excitation; Importance sampling; Reliability-based optimization; Uncertain systems

1. Introduction

Structural optimization via general nonlinear mathematical programming techniques has been widely accepted as a viable tool for engineering design. When a structure is being designed the environmental loads that the built structure will experience in its lifetime are highly uncertain. The load time history needed in the dynamic analysis of a structure subject to environmental loads such as wind, water wave excitation, and earthquake is an uncertain value function, and it is best modeled by a stochastic process [15,26,18,5]. Likewise, response predictions are made during the design stage based on structural models whose parameters are uncertain. This is because the properties that will be exhibited by the structure when completed are not known precisely. These uncertainties result from the numerous assumptions made when modeling the geometry, boundary conditions, material behavior, etc. Probabilistic methods provide the means for incorporating system uncertainties as random variables with a prescribed joint probability density function. Uncertainties in both loading and structural properties can adversely affect the reliability and performance of the structural system. Therefore, it is necessary to consider their effects explicitly during the optimization process to achieve a balance between cost and safety for the optimal design.

In reliability-based structural optimization, the constraints are usually reliability requirements with respect to possible failure modes of the structure. Probability that design conditions are satisfied within a given time period is commonly used as a measure of system reliability [6,19,24]. Then, the first excursion probability that any one of the system response functions of interest exceeds in magnitude some specified threshold level within a given time duration needs to be estimated. In an optimization environment,

* Corresponding author.
E-mail address: hector.jensen@usm.cl (H.A. Jensen).

0045-7825/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.cma.2008.01.003

reliability measures need to be evaluated several times before a near optimal design can be obtained. System reliability that accounts for the uncertainties in the system parameters as well as the uncertainty in the excitation is given by the total probability theorem. It takes the form of a particular multidimensional integral over the space of uncertain system parameters. For complex systems the estimate of reliability measures at a given design must be obtained by carrying out a simulation procedure [22]. Then, the evaluation of these quantities for every change of the optimization variables requires the evaluation of dynamic responses of the structural system. In general, these responses are nonlinear implicit functions of the design variables and uncertain system parameters. For systems of practical interest, the repeated evaluation of dynamic responses can be extremely time consuming. From an optimization point of view, reliability-based optimization problems can be characterized as two-level optimization problems. Level one is the overall optimization in the design variables, and level two is the reliability estimates. For realistic systems, these estimates completely dominate the total calculation cost. Therefore, the reliability estimates should be evaluated in an efficient manner and the number of response calculations must be as few as possible during the optimization process.

The general formulation of reliability-based structural optimization may involve a wide range of structural problems such as linear and nonlinear systems with uncertainties in the material properties and/or loading conditions. This paper focuses on a particular class of important problems in stochastic dynamics. Namely, linear systems with uncertain structural parameters subjected to general Gaussian excitation. The specific purpose of this work is to develop an efficient computational procedure for the reliability-based optimization of uncertain stochastic linear dynamical systems. The proposed methodology can be seen as a further development of the method presented in [12]. In that work, an efficient procedure which allows to carry out reliability-based optimization of deterministic linear systems under stochastic loading was presented. An approximation strategy was introduced to construct high quality approximations of dynamic responses. The approximations were combined with an efficient simulation technique to generate explicit approximations of reliability measures in terms of the design variables. In the present approach, the system reliability is estimated by an efficient simulation technique that accounts for the uncertainty in the system parameters and excitation. The technique is combined with an efficient local approximation strategy for the approximation of reliability measures in terms of the design variables. The proposed procedure dramatically reduces the number of system analyses as well as the reliability estimations required during the entire design process.

First, the definition of the structural optimization problem and the corresponding mechanical modeling of the physical system are presented. An efficient methodology to estimate excursion probabilities of uncertain linear systems subjected to stochastic excitations is then discussed. Next, an effective approximation strategy to carry out the optimization process is considered. Finally, some example problems are presented to illustrate the performance of the proposed method.

2. Formulation of the optimization problem

Let the vectors $\{y\}$, $y_i, i = 1, \ldots, n_d$, $\{\theta\}$, θ_i, $i = 1, \ldots, n_s$, and $\{z\}$, z_i, $i = 1, \ldots, n_T$ represent the vector of design variables, uncertain structural parameters, and random variables that specify the stochastic excitation, respectively. The uncertain structural parameters $\{\theta\}$ are modeled using a prescribed probability density function $q(\{\theta\})$ while the random variables $\{z\}$ are characterized by a probability function $p(\{z\})$. These functions indicate the relative plausibility of the possible values of the uncertain parameters $\{\theta\} \in \Omega_{\{\theta\}} \subset R^{n_s}$ and random variables $\{z\} \in \Omega_{\{z\}} \subset R^{n_T}$, respectively. In this context, it is clear that a system response r_i evaluated at the design $\{y\}$ depend on the particular set of values $(\{\theta\}, \{z\}) = [\theta_1, \ldots, \theta_{n_s}, z_1, \ldots, z_{n_T}]$ of the set of uncertain parameters $(\{\Theta\}, \{Z\}) = [\Theta_1, \ldots, \Theta_{n_s}, Z_1, \ldots, Z_{n_T}]$ that may assume, that is $r_i(t, \{y\}, \{\theta\}, \{z\})$. The structural synthesis problem considered in the present formulation is written as

$$\text{Min}_{\{y\}} \left\{ C(\{y\}) | P_F(\{y\}) \leqslant P_F^{\text{accept}} \right\}, \quad \{y\} \in Y, \tag{1}$$

where $Y \subset R^{n_d}$ is the set that contains the side constraints for the design variables, $C(\{y\})$ is an objective function which is assumed to be an explicit function of the design variables, $P_F(\{y\})$ is the failure probability, and P_F^{accept} is the target system reliability. Then, the problem consists in the determination of a set of design variables that minimizes an objective function for a specified reliability. The failure probability function $P_F(\{y\})$ accounts for the uncertainties in the system parameters as well as the uncertainties in the excitation. The failure probability is given in terms of the probability that some stochastic dynamic responses exceed in magnitude within a specified time interval $[0, T]$ certain critical threshold levels. That is, $P_F(\{y\}) = P[F]$, where the failure event F is characterized by

$$F = \left\{ \max_{i=1,\ldots,n_r} \max_{t \in [0,T]} |r_i(t, \{y\}, \{\theta\}, \{z\})| > r_i^* \right\}, \tag{2}$$

and where $P[\cdot]$ is the probability that the expression in parenthesis is true, $r_i(t, \{y\}, \{\theta\}, \{z\})$, $i = 1, \ldots, n_r$ are the response functions of interest, and $r_i^*, i = 1, \ldots, n_r$ are the threshold levels. In terms of the joint probability density function, the failure probability can be written as a classical probability integral:

$$P_F(\{y\}) = \int_{\Omega_{\{\theta\}}} \int_{\Omega_{\{z\}}} \Pi_F(\{y\}, \{\theta\}, \{z\}) q(\{\theta\}) p(\{z\}) \, d(\{z\}) d(\{\theta\}) \tag{3}$$