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An efficient framework for optimal robust stochastic system design using stochastic simulation

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ABSTRACT

The knowledge about a planned system in engineering design applications is never complete. Often, a probabilistic quantification of the uncertainty arising from this missing information is warranted in order to efficiently incorporate our partial knowledge about the system and its environment into their respective models. This leads to a robust stochastic design framework where probabilistic models of excitation uncertainties and system modeling uncertainties can be introduced; the design objective is then typically related to the expected value of a system performance measure, such as reliability or expected life-cycle cost. For complex system models, this expected value can rarely be evaluated analytically and so it is often calculated using stochastic simulation techniques, which involve an estimation error and significant computational cost. An efficient framework, consisting of two stages, is presented here for the optimization in such robust stochastic design problems. The first stage implements a novel approach, called stochastic subset optimization (SSO), for iteratively identifying a subset of the original design space that has high plausibility of containing the optimal design variables. The second stage adopts some other stochastic optimization algorithm to pinpoint the optimal design variables within that subset. The focus is primarily on the theory and implementation issues for SSO but also on topics related to the combination of the two different stages for overall enhanced efficiency. An illustrative example is presented that shows the efficiency of the proposed methodology; it considers the optimization of the reliability of a base-isolated structure considering future near-fault ground motions.

1. Introduction

In engineering design, the knowledge about a planned system is never complete. First, it is not known in advance which design will lead to the best system performance in terms of a specified metric; it is therefore desirable to optimize the performance measure over the space of design variables that define the set of acceptable designs. Second, modeling uncertainty arises because no mathematical model can capture perfectly the behavior of a real system and its environment (future excitations). In practice, the designer chooses a model that he or she feels will adequately represent the behavior of the built system as well as its future excitation; however, there is always uncertainty about which values of the model parameters will give the best representation of the constructed system and its environment, so this parameter uncertainty should be quantified. Furthermore, whatever model is chosen, there will always be an uncertain prediction error between the model and system responses. For an efficient engineering design, all uncertainties, involving future excitation events as well as the modeling of the system, must be explicitly accounted for. A probability logic approach provides a rational and consistent framework for quantifying all of these uncertainties [1]. In this case, this process may be called *robust stochastic system design*.

In this context, consider some controllable parameters that define the system design, referred to herein as *design variables*, $\boldsymbol{\varphi} = [\varphi_1 \varphi_2, \ldots, \varphi_{n_\varphi}] \in \Phi \subset \mathbb{R}^{n_\varphi}$, where Φ denotes the bounded admissible design space. Also consider a model class that is chosen to represent a system design and its future excitation, where each model in the class is specified by an n_θ-dimensional vector $\boldsymbol{\theta} = [\theta_1 \theta_2 \ldots \theta_{n_\theta}]$ lying in $\Theta \subset \mathbb{R}^{n_\theta}$, the set of possible values for the model parameters. Because there is uncertainty in which model best represents the system behavior, a PDF (probability density function) $p(\boldsymbol{\theta}|\boldsymbol{\varphi})$, which incorporates available knowledge about the system, is assigned to these parameters. The performance for a robust-to-uncertainties design is, then, expressed by the stochastic integral:

$$E_{\boldsymbol{\theta}}[h(\boldsymbol{\varphi}, \boldsymbol{\theta})] = \int_{\Theta} h(\boldsymbol{\varphi}, \boldsymbol{\theta}) p(\boldsymbol{\theta}|\boldsymbol{\varphi}) \mathrm{d}\boldsymbol{\theta}, \quad (1)$$

where $E_{\boldsymbol{\theta}}[\cdot]$ denotes expectation with respect to the PDF for $\boldsymbol{\theta}$ and $h(\boldsymbol{\varphi}, \boldsymbol{\theta}) : \mathbb{R}^{n_\varphi} \times \mathbb{R}^{n_\theta} \to \mathbb{R}$ denotes the performance measure of the system. In engineering applications, stochastic design problems are many times posed by adopting deterministic objective functions

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and using constraints related to stochastic integrals like (1) to characterize the admissible design space-such an approach is common, for example, in the context of reliability-based design optimization (RBDO) where reliability constraints are adopted [2,3]. In this study, though, we focus on design problems that entail as objective function a stochastic integral of the form (1). The *optimal stochastic design problem* in this case takes the form:

$$\begin{aligned} &\text{minimize} \quad E_{\boldsymbol{\theta}}[h(\boldsymbol{\varphi},\boldsymbol{\theta})],\\ &\text{subject to} \quad \mathbf{f}_c(\boldsymbol{\varphi}) \geqslant 0, \end{aligned} \tag{2}$$

where $\mathbf{f}_c(\boldsymbol{\varphi})$ corresponds to a vector of constraints. Such optimization problems, arising in decision making under uncertainty, are typically referred to as stochastic optimization problems (e.g. [4,5]). The constraints in optimization (2) can be taken into account by appropriate definition of the admissible design space Φ; the stochastic design problem is then equivalently formulated as:

$$\boldsymbol{\varphi}^* = \arg\min_{\boldsymbol{\varphi}\in\Phi} E_{\boldsymbol{\theta}}[h(\boldsymbol{\varphi},\boldsymbol{\theta})]. \tag{3}$$

For this optimization, the integral in (1) must be evaluated. For complex systems this integral can rarely be calculated, or even efficiently approximated, analytically and so it is commonly evaluated through stochastic simulation techniques. In this setting, an unbiased estimate of the expected value in (1) can be obtained using a finite number, N, of random samples of $\boldsymbol{\theta}$, drawn from $p(\boldsymbol{\theta}|\boldsymbol{\varphi})$:

$$\widehat{E}_{\boldsymbol{\theta},N}[h(\boldsymbol{\varphi},\boldsymbol{\Omega}_N)] = \frac{1}{N}\sum_{i=1}^{N} h(\boldsymbol{\varphi},\boldsymbol{\theta}_i), \tag{4}$$

where $\boldsymbol{\Omega}_N = [\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_N]$ is the sample set of the model parameters with vector $\boldsymbol{\theta}_i$ denoting the sample of these parameters used in the ith simulation. This estimate of $E_{\boldsymbol{\theta}}[h(\boldsymbol{\varphi},\boldsymbol{\theta})]$ involves an unavoidable error $e_N(\boldsymbol{\varphi},\boldsymbol{\Omega}_N)$ which is a complex function of both the sample set $\boldsymbol{\Omega}_N$ as well as the current system model configuration. The optimization in (3) is then approximated by:

$$\boldsymbol{\varphi}_N^* = \arg\min_{\boldsymbol{\varphi}\in\Phi} \widehat{E}_{\boldsymbol{\theta}}[h(\boldsymbol{\varphi},\boldsymbol{\Omega}_N)]. \tag{5}$$

If the stochastic simulation procedure is a consistent one, then as $N\to\infty$, $\widehat{E}_{\boldsymbol{\theta},N}[h(\boldsymbol{\varphi},\boldsymbol{\Omega}_N)] \to E_{\boldsymbol{\theta}}[h(\boldsymbol{\varphi},\boldsymbol{\theta})]$ and $\boldsymbol{\varphi}_N^* \to \boldsymbol{\varphi}^*$ under mild regularity conditions for the optimization algorithms used [5]. The existence of the estimation error $e_N(\boldsymbol{\varphi}, \boldsymbol{\Omega}_N)$, which may be considered as noise in the objective function, contrasts with classical deterministic optimization where it is assumed that one has perfect information. Fig. 1 illustrates the difficulties associated with $e_N(\boldsymbol{\Omega}_N,\boldsymbol{\varphi})$. The curves corresponding to simulation-based evaluation of the objective function have non-smooth characteristics, a feature which makes application of gradient-based algorithms challenging. Also, the estimated optimum depends on the exact influence of the estimation error, which is not the same for all evaluations. Another source of difficulty, especially when complex system models are considered, is the high computational cost associated with the estimation in (5) since N system analyses must be performed for each objective function evaluation. Even though recent advanced stochastic optimization algorithms (see Section 3) can efficiently address the first two aforementioned problems this latter one remains challenging for many engineering design applications. Specialized, approximate approaches have been proposed in various engineering fields for reduction of the computational cost (e.g. [2,3,6] for RBDO problems). These approaches may work satisfactorily under certain conditions, but are not proved to always converge to the solution of the original design problem. For this reason such approaches are avoided in this current study. Optimization problem (5) is directly solved so that $\boldsymbol{\varphi}_N^* \approx \boldsymbol{\varphi}^*$.

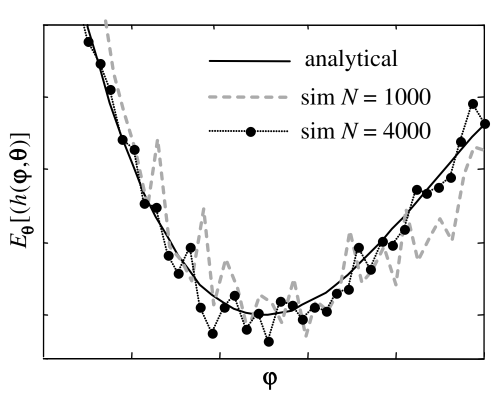

Fig. 1. Analytical and simulation-based (sim) evaluation of an objective function.

An efficient framework, consisting of two stages, is discussed in the following sections for a computational efficient solution to this optimization. The first stage implements a novel approach, called Stochastic subset optimization (SSO) [7,8], for efficiently exploring the global sensitivity of the objective function to the design variables and for iteratively converging to a subset of the original design space that has high plausibility of containing the optimal design variables and, additionally, consists of near-optimal design configurations. The second stage adopts some appropriate stochastic optimization algorithm to pinpoint, more precisely, the optimal design variables within the set identified in the first stage. The focus is primarily on the theory and implementation issues for SSO but also on topics related to the combination of the two different stages for overall enhanced efficiency.

2. Stochastic subset optimization

Stochastic subset optimization (SSO) was initially suggested for reliability-based optimization problems (for a proper definition of such problems see Section 5.1 later on) in [9] and has been recently [8] extended to address general stochastic design problems, such as the one in (2). The basic features of the algorithm are summarized next.

2.1. Augmented problem and subset analysis

Consider the modified positive function, $h_s(\boldsymbol{\varphi},\boldsymbol{\theta}): \mathbb{R}^{n_\varphi}\times\mathbb{R}^{n_\theta}\to\mathbb{R}^+$ defined as

$$h_s(\boldsymbol{\varphi},\boldsymbol{\theta}) = h(\boldsymbol{\varphi},\boldsymbol{\theta}) - s \quad \text{where } s < \min_{\boldsymbol{\varphi},\boldsymbol{\theta}} h(\boldsymbol{\varphi},\boldsymbol{\theta}) \tag{6}$$

and note that $E_{\boldsymbol{\theta}}[h_s(\boldsymbol{\varphi},\boldsymbol{\theta})] = E_{\boldsymbol{\theta}}[h(\boldsymbol{\varphi},\boldsymbol{\theta})] - s$. Since the two expected values differ only by a constant, optimization of the expected value of $h(\cdot)$ is equivalent, in terms of the optimal design choice, to optimization for the expected value for $h_s(\cdot)$. In the SSO setting we focus on the latter optimization.

The basic idea in SSO is the formulation of an augmented problem, a general concept initially discussed in [10] for reliability-based design problems, where the design variables are artificially considered as uncertain with distribution $p(\boldsymbol{\varphi})$ over the design space Φ. In the setting of this augmented stochastic design problem, define the auxiliary PDF:

$$\pi(\boldsymbol{\varphi},\boldsymbol{\theta}) = \frac{h_s(\boldsymbol{\varphi},\boldsymbol{\theta})p(\boldsymbol{\varphi},\boldsymbol{\theta})}{E_{\boldsymbol{\varphi},\boldsymbol{\theta}}[h_s(\boldsymbol{\varphi},\boldsymbol{\theta})]} \propto h_s(\boldsymbol{\varphi},\boldsymbol{\theta})p(\boldsymbol{\varphi},\boldsymbol{\theta}), \tag{7}$$

where $p(\boldsymbol{\varphi},\boldsymbol{\theta}) = p(\boldsymbol{\varphi})p(\boldsymbol{\theta}|\boldsymbol{\varphi})$. The normalizing constant in the denominator is defined as:

$$E_{\boldsymbol{\varphi},\boldsymbol{\theta}}[h_s(\boldsymbol{\varphi},\boldsymbol{\theta})] = \int_\Phi\int_\Theta h_s(\boldsymbol{\varphi},\boldsymbol{\theta})p(\boldsymbol{\varphi},\boldsymbol{\theta})\,d\boldsymbol{\theta}\,d\boldsymbol{\varphi} \tag{8}$$

and corresponds to the expected value in the augmented uncertain space. This expected value is not explicitly needed, but it can be obtained though stochastic simulation, which leads to an expression