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Review

Inhomogeneous, orthotropic material model for the cortical structure of long bones modelled on the basis of clinical CT or density data

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ABSTRACT

For the simulation of stabilisation systems in femoral fractures with finite elements three factors are essential: (1) the geometry of the bone, (2) the loading of the acting muscle forces in combination with the body weight and (3) the inhomogeneously distributed orthotropic behaviour of the bone material must be known.

This study will focus on the third condition.

A technique is presented for transferring the density distributions gained from clinical computer tomographies to inhomogeneous, orthotropic material distributions suitable for finite element calculations.

First, the algorithm for determining the orthotropy directions from the local variation of the density field is explained phenomenologically. Subsequently, a function for setting up the orthotropic elasticity matrices from the absolute CT field values is derived. Finally, the validation procedure for the cortical bone is presented in principle.

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1. Introduction

As Wolff stated in his study "Ueber die innere Architectur der Knochen und ihre Bedeutung für die Frage von Knochenwachsthum" [About the inner architecture of bone and its meaning for the question of bone growth] [1] as early as 1870, bone or bone material is subject to continuous modification processes. It reacts to altered load situations by modifying itself. An implantation, which massively modifies the distribution of forces in the bone, can have both a positive as well as a negative effect on the modification process and on the healing of the bone segments created by a fracture.

With the aid of modern imaging techniques such as computer tomography (CT), it is possible nowadays to gain relatively precise information on the geometrical structure of a bone, varying as it does from one person to another.

In order to gain reliable results regarding the altered force flow with the finite element method, however, two further conditions must be fulfilled. On the one hand, the effects of the body weight and the muscles on, for example, the femur must be known. On the other hand, a material model is required which realistically reproduces the material constants of the bone as well as the directions they are working towards.

The CT provides an image of the highly individual internal composition of the bone material which is closely connected to the material properties of the bone via the scalar field of the density.

In order to render the calculation of the stabilisation system in medicine into a useful and reliable tool, it is necessary to resort to the CT data as they often provide the only information which can be gained from a living bone.

Many recent studies on this subject [2–4] show that it is indeed possible to derive inhomogeneous, isotropic material properties from these data.

A series of further studies, for example [5–7], shows that the material properties of bone can be represented by an orthotropic material formulation.

The aim of the present study is to combine these two factors and to develop a procedure, on the basis of the data gained from clinical CT images, which can set up an orthotropic, inhomogeneous material distribution and transfer it automatically to a finite element model.

The first section of the study presents the fundamentals of orthotropic material behaviour. The procedure developed here is divided into two parts because of the information required for an orthotropic material model, i.e. the elasticity matrix and the appertaining orthotropy directions; the second section presents the algorithm which delivers the information on the directions required for the orientation of the orthotropic symmetry planes by analysing the variation – or rather smoothness – of the density field. In the third section, a function is developed which makes the elasticity matrix dependent on the CT values. This function is parameter-dependent and is based on the relationships taken from the literature between orthotropic Young's moduli and CT data.

2. Orthotropic material behaviour

For orthotropic material behaviour, the general strain–stress relation simplifies to

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{13} & s_{23} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} \qquad (1)$$

in Voigt notation.

The compliance matrix in Eq. (1) is formed by three orthogonal symmetry planes which characterise the orthotropic material behaviour [9]. If the load on the orthotropic material is normal to one of these symmetry planes, then only normal strains and no shear strains occur. This means that, in the loading case, normal and shear strains are decoupled in the orthotropy directions.

As can be seen in Eq. (1), the symmetry of the compliance matrix means that nine constants must be given to describe orthotropic material behaviour.

As an alternative to Eq. (1), the coefficients of the compliance matrix can also be expressed in simple relations of the material constants,

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & -\frac{\nu_{32}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix}. \qquad (2)$$

Evidently, in this representation, 12 material constants are used to describe the orthotropic material behaviour. However, it is necessary to observe the symmetry conditions of the compliance matrix

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1}, \qquad (3)$$

$$\frac{\nu_{31}}{E_3} = \frac{\nu_{13}}{E_1}, \qquad (4)$$

$$\frac{\nu_{32}}{E_3} = \frac{\nu_{23}}{E_2}, \qquad (5)$$

or, in index notation,

$$\frac{\nu_{ji}}{E_j} = \frac{\nu_{ij}}{E_i}, \qquad (6)$$

where $i = 1, 2$, $j = 2, 3$ and $i \neq j$.

These relations indicate, that out of the 12 material constants only nine are independent.

Since the compliance matrix takes the form of Eqs. (1) and (2) only in the case of alignment along the orthotropy directions, two orthogonal vectors are also necessary in order to describe the local position of the orthotropic symmetry directions.

3. Orientation

3.1. Orthotropy directions within the bone material

If a cylindrical coordinate is introduced in the diaphysis section of a long bone, then, as can be seen in Fig. 1, a local tangential coordinate system can be established. In this coordinate system, direct relations between the absolute values of the three orthotropic

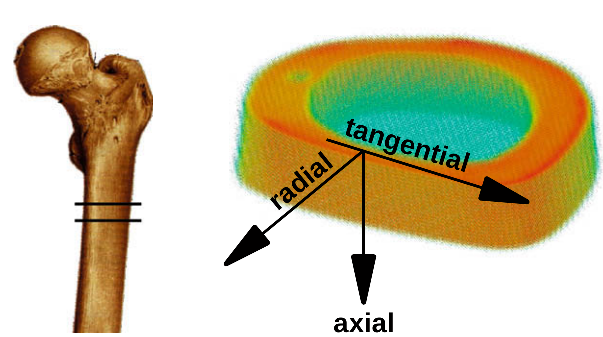

Fig. 1. Local coordinate system in the diaphysis.