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Solution of the boundary-layer equations for natural convection film boiling on vertical cylindrical surfaces

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A R T I C L E I N F O

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A B S T R A C T

Film boiling around a cylinder has been studied with a pseudo-similarity method. The velocity, temperature profile, the heat and mass transfer performance and the vapor film thickness could be predicted by the model with the effects of thermal properties (Pr, vapor liquid $\rho\mu$ ratio, and $\frac{c_{pv}(T_w - T_{sat})}{h_{lv}\text{Pr}}$) considered. The results show that the heat transfer performance of film boiling on a cylinder surface is much higher than that on a flat surface. The film temperature and the Nusselt number of the numerical model agree well with the experimental data from the literature within the tested ranges. The heat transfer rate and local heat flux of the pseudo-similarity method in this work are also compared with that of the perturbation method, the results from those two method are found a perfect match.

1. Introduction

Film boiling heat transfer has been a subject of study for many years. Research on this subject has been focused on laminar film boiling or condensation on a vertical or horizontal plate, vertical or horizontal cylinders, and film boiling from submerged spheres. The pioneer theoretical work was reported by Koh [1], who treated the problem by taking account of shear stress and vapor velocity at the vapor–liquid interface. Laminar natural convection along the outer surface of a vertical cylinder is compared with that along a vertical flat plate on heat transfer was conducted by Fujii and Uehara [2]. In their study, the perturbation solution of the boundary layer gives two empirical expressions for heat transfer coefficient those relate flat case to cylinder case. Experiments by Shiotsu and Hama [3] revealed the heat transfer coefficient in forced convection for a vertical cylinder to be independent of its position up to a critical value of the liquid velocity. Das et al. [4] conducted a simulation study based on scale analysis, the model takes care of natural convection and forced convection mode of heat transfer for subcooled case, while the saturated film boiling is treated with an assisting convection mode. The effect of radiation heat transfer is accounted in the model. Expressions for Nusselt number have been developed for different situations. Meduri et al. [5] worked with a vertical plate under subcooled forced convection and developed a generalized correlation from their experimental data. Based on the newly developed phase-change Lattice Boltzmann method,the problem of steady laminar film condensation on a hydrophilic vertical flat plate at a subcooled temperature was simulated by Liu and Cheng [6]. The condensate film thickness, velocity and temperature distributions, and heat transfer characteristics were obtained numerically, however the model can only be carried out for the initial condition of saturated vapor. Begmohammadi et al. [7] extend the dynamic model of Liu to investigate film boiling numerically based on the Lattice Boltzmann method. The phase-change process is modeled by incorporating a proper source term at the phase interface. For the diversity of different shape, Son and Dhir [8] solved the equations of the horizontal cylinder governing the conservation of mass, momentum and energy in vapor and liquid phases. A level set formulation for tracking the liquid–vapor interface is modified to include the effect of phase change at the liquid–vapor interface and to treat the no-slip condition at the fluid–solid interface. It should be noted that most of previous analytical studies are concentrated on the film boiling or condensation on a flat plate or a horizontal tube, whereas many industrial evaporating systems employ vertical tube evaporators recently. In this study, numerical simulations of film boiling on a vertical cylinder are performed on the basis of integral-transformation method. The distribution for the velocity profiles as well as local and average Nusselt numbers and many other parameters are derived. The model was developed in cylindrical coordinate with concern of the flow pattern of the film in reality, which is ignored in most previous work. Bottom surface effect was also considered in the model. The main factors influencing the heat transfer performance were

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Nomenclature

u	velocity component in x-direction
v	velocity component in r-direction
x	coordinate measuring distance along cylinder from the leading edge
r	coordinate measuring distance normal to cylinder surface
g	acceleration due to gravity
T	temperature
P	pressure
h_{lv}	latent heat of evaporation
C	dimensional constants
Nu_x	local Nusselt number
f	dimensionless velocity variable for vapor
q	local heat-transfer rate per unit area from wall to vapor
c_{pv}	specific heat of vapor at constant pressure
$\dot{M}$	local mass flow rate of vapor at the bottom corner
y	position normal to the bottom surface
D	diameter of cylinder
α	thermal diffusivity
δ	thickness of vapor layer
μ	dynamic viscosity
ν	kinematic viscosity
k	thermal conductivity
ρ	density
θ	dimensionless temperature
η	similarity variable relate to vapor phase
ξ	similarity variable relate to liquid phase
v	vapor
l	liquid
w	wall
δ	wall liquid interface
sat	saturated condition
e	equilibrium state
A	horizontal bottom surface

calculated and analyzed.

2. Model and equations

2.1. Governing equations

A physical model of the laminar film boiling on the outer surface of a vertical cylinder is described in Fig. 1. All relevant variables are defined in cylindrical coordinates. An isothermal vertical cylinder is suspended in a large volume of liquid. The liquid is at its saturation temperature $T_{\mathrm{sat}}(P_l)$. The cylinder temperature T_w is higher than $T_{\mathrm{sat}}(P_l)$ and therefore vaporization occurs on the cylinder surface. Film boiling under the bottom surface may effect the heat transfer rate on the vertical lateral surface of the body. The vapor produced beneath the horizontal bottom surface flows upward from the edge of the horizontal bottom surface and results in thickening the vapor film that coats the vertical lateral surface of the body.

The governing equations for the conservation of mass, momentum and energy for steady laminar flow in a vapor boundary layer on a vertical cylinder are expressed as follows:

Continuity
$$\frac{\partial (r u_v)}{\partial x} + \frac{\partial (r v_v)}{\partial r} = 0. \tag{1}$$

Momentum
$$u_v \frac{\partial u_v}{\partial x} + v_v \frac{\partial u_v}{\partial r} = g\beta (T_v - T_{\mathrm{sat}}) + \nu_v \left(\frac{\partial^2 u_v}{\partial r^2} + \frac{1}{r}\frac{\partial u_v}{\partial r} \right). \tag{2}$$

Energy
$$u_v \frac{\partial T_v}{\partial x} + v_v \frac{\partial T_v}{\partial r} = \alpha_v \left(\frac{\partial^2 T_v}{\partial r^2} + \frac{1}{r}\frac{\partial T_v}{\partial r} \right). \tag{3}$$

The fluid properties are those of the vapor phase. Viscous dissipation has been neglected, the same for the temperature dependence of the fluid properties. In this analysis, transport of heat across the vapor film by radiation is also neglected.

For the liquid adjacent to the vapor film, the temperature is assumed to be equal to $T_{\mathrm{sat}}(P_l)$ everywhere, since the liquid in the ambient pool is at the saturate temperature. Consistent with the usual boundary-layer approximations, the motion of the liquid is assumed to be governed by the following forms of the continuity and momentum equations:

Continuity
$$\frac{\partial (r u_l)}{\partial x} + \frac{\partial (r v_l)}{\partial r} = 0. \tag{4}$$

Momentum
$$u_l \frac{\partial u_l}{\partial x} + v_l \frac{\partial u_l}{\partial r} = \nu_l \left(\frac{\partial^2 u_l}{\partial r^2} + \frac{1}{r}\frac{\partial u_l}{\partial r} \right). \tag{5}$$

The boundary conditions at the heated wall ($r = R$), at the liquid–vapor interface ($r = R + \delta$), and in the far ambient ($r \to \infty$) are at $r = R$ (vapor phase):

$$u_v = 0, \quad v_v = 0, \quad T_v = T_w \tag{6}$$

at $r = R + \delta$ (liquid–vapor interface)

$$u_v = u_l, \quad \mu_v \left(\frac{\partial u_v}{\partial r} \right) = \mu_l \left(\frac{\partial u_l}{\partial r} \right) \tag{7}$$

$$\rho_v u_v \frac{\partial \delta_v}{\partial x} - \rho_v v_v = \rho_l u_l \frac{\partial \delta_l}{\partial x} - \rho_l v_l \tag{8}$$

$$T = T_e = T_{\mathrm{sat}}(P_l) \tag{9}$$

as $r \to \infty$ (Liquid phase)

$$u_l \to 0, \quad v_l \to 0. \tag{10}$$

At the wall, the boundary conditions are a consequence of the no-slip condition and the isothermal wall specification. The relations Eqs. (7) and (9) that apply for $r = R + \delta$ specify continuity of the u-velocity, shear stress, and the temperature profile across the interface. Eq. (8) imposes mass conservation on a different control volume at the curved interface, as indicated in Fig. 2.

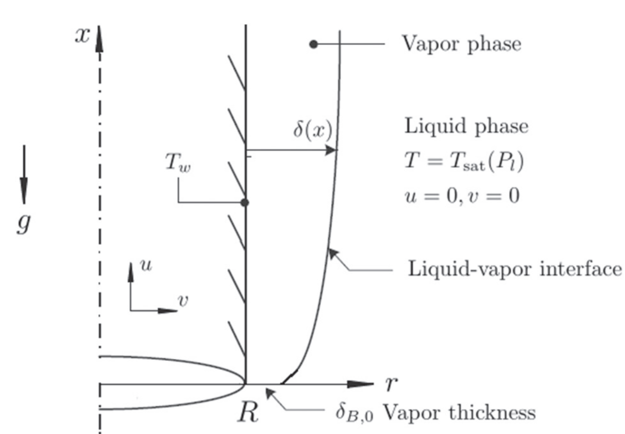

Fig. 1. Laminar film boiling on a vertical cylinder.