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Inverse alloy solidification problem including the material shrinkage phenomenon solved by using the bee algorithm

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ABSTRACT

In the paper we solve the one-phase inverse problem of alloy solidifying within the casting mould, including the shrinkage of metal which results from the difference between densities of the liquid and solid phases. The process is modeled by means of the solidification in the temperature interval basing on the heat conduction equation with the source element enclosed, whereas the shrinkage of metal is modeled by the proper application of the mass balance equation. The investigated inverse problem consists in reconstruction of the heat transfer coefficient on the boundary of the casting mould on the basis of measurements of temperature read from the sensor placed in the middle of the mould. Functional expressing the error of approximate solution is minimized with the aid of Artificial Bee Colony Optimization algorithm.

1. Introduction

The inverse problem is a task of reconstructing some parameters of the model on the basis of observations made on the modeled process [1–5]. It is called an inverse problem because, while solving it, we start with the results and we intend to determine the causes. This kind of problem is therefore very important from the practical point of view because it gives the possibility to control the investigated process, that is to select the parameters influencing the run of the process to achieve the product in the expected form and of the required quality. Authors of this paper considered already the inverse problems connected with the heat conduction. Papers [6–8] present some methods developed for solving the two-dimensional inverse Stefan problem and the three-dimensional inverse continuous casting process.

In the inverse problems considered by us till now we assumed the perfect contact between the cast and the casting mould. However in the real processes, due to the difference between densities of the liquid and solid phases, the shrinkage of metal often appears during the solidification. Then, the air-gap may form between the cast and the casting mould. The created air-gap generates in turn the interfacial thermal resistance between the mould and the cast determining the mould heat flux which, in result, decreases the quality of the product causing some defects, such as the cracks or oscillation marks [9]. Therefore, it is very important to undertake some efforts leading to control this phenomenon.

Nawrat and Skorek in papers [10,11] investigated the heat resistance of the air gap created between the ingot and crystallizer in the continuous casting process. For modeling the solidification process they used the Stefan problem and they determined the heat conduction coefficient of the gap on the basis of temperature measurements in the crystallizer walls. In works [12,13] the heat resistance of the gap between the mould or the crystallizer and the ingot was also determined, whereas in papers [14,15] the interfacial heat transfer coefficient between the form and the cast was computed. Authors of the current paper investigated already the phenomenon of metal shrinkage in the solidification process in papers [16–18]. In the first two works the solidification of the pure metal was considered and modeled by means of the one-dimensional Stefan problem, whereas in [18] the solution technique for solving the direct problem of the alloy solidifying within the casting mould was tested. The investigated process was modeled there by means of the solidification in the temperature interval basing on the heat conduction equation with the source element enclosed, which includes the latent heat of fusion and the volume contribution of solid phase [19–22], whereas the shrinkage of metal was modeled by the proper application of the mass balance equation. The model of solidification in the temperature interval assumes the existence of three states in the solidifying material, that is the liquid phase, solid phase and the intermediate zone (called the mushy zone) between them, where the both phases coexist [23]. Therefore this model is often used for modeling the solidification of alloys, in contrast to the Stefan model applied for modeling the solidification of pure metals, in which the liquid and solid phases are sharply separated by the interphase surface.

Similar approach as in [18] is applied in the current paper. We examine here the one-phase inverse problem of alloy solidifying within

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Nomenclature

b	Length
c_l	Specific heat of the liquid phase
c	Specific heat
C	Substantial thermal capacity
D	Dimension of minimized problem
d	Thickness of the plate
f_s	Volumetric solid state fraction
h	Height of the plate
ΔH_i	Change of enthalpy
J	Minimized functional
l	Width of the plate
L	Latent heat of solidification
lim	Number of corrections of the source location
m	Mass of the alloy
MCN	Maximal number of cycles
N	Number of measurements
P	Probability of choosing the source of nectar
R	Thermal resistance
SN	Number of bees (sources of nectar)
t	Time
t^*	Final time
T	Temperature
T_L	Liquidus temperature
T_S	Solidus temperature
T_∞	Ambient temperature
U_i	Measured temperature
v_x	Velocity vector
V_j	Control volume
x	Spatial variable
$\mathbf{x}_{best}$	Best located bee

Greek symbols

α	Heat transfer coefficient
δ	Relative percentage error
Δ	Absolute error
λ	Thermal conductivity
ρ	Mass density
$\xi_{l(s)}$	Location of $T_{L(S)}$
Ω	Considered region

Subscripts

l	Liquid phase
m	Mould
mz	Mushy zone
s	Solid phase
0	Initial

the casting mould including the change of cast size caused by the material shrinkage. The investigated inverse problem consists in reconstruction of the heat transfer coefficient on the boundary of the casting mould on the basis of measurements of temperature read from the sensor placed in the middle of the mould. Solution of the corresponding direct problem is technically based on the finite difference method supplemented by the procedure of correcting the field of temperature in the vicinity of the liquidus and solidus curves [21,22,24]. Next, on the ground of the measurement values the functional expressing the error of approximate solution is formulated and minimized with the aid of Artificial Bee Colony Optimization algorithm [25–27] in order to select the value of sought heat transfer coefficient so that the reconstructed values of temperature approximate as best as possible the measured values.

2. Formulation of the problem

We consider the solidification of a plate of the following dimensions: thickness $d(t)$, width h and height l (we assume $d(t) \ll h$ and $d(t) \ll l$). The solidifying material occupies region $\Omega = \{(x,t) : x \in (0,d(t)), t \in (0,t^*)\}$ divided into three subregions: taken by the solid phase, liquid phase and the mushy zone where the both phases coexist. The cast is bounded by the casting mould, the region of which is denoted by $\Omega_m = \{(x,t) : x \in (d_0,b), t \in (0,t^*)\}$. While the solidification process proceeds, the air gap creates between the cast and the casting mould. Thus, in the initial moment $d(0) = d_0$ and next, the cast boundary $d(t)$ moves and forms the gap. The scheme of investigated situation is presented in Fig. 1.

Neglecting the natural convection in the liquid phase, as well as the strain energy of the mushy zone, we assume that the temperature field in region Ω is described by the following heat conduction equation

$$c\,\rho\frac{\partial T(x,t)}{\partial t} + v_x\frac{\partial T(x,t)}{\partial x} = \lambda\frac{\partial^2 T(x,t)}{\partial x^2} + L\,\rho\frac{\partial f_s(x,t)}{\partial t}, \tag{1}$$

for $(x,t) \in \Omega$, where c, ρ and λ are the specific heat, mass density and thermal conductivity coefficient, respectively, v_x means the velocity vector, L denotes the latent heat of solidification, f_s describes the volumetric solid phase fraction, T is the temperature, and finally, t and x refer to the time and spatial variables. The volumetric solid state fraction depends on the temperature, so we may write

$$\frac{\partial f_s}{\partial t} = \frac{\partial f_s}{\partial T}\frac{\partial T}{\partial t}. \tag{2}$$

Substituting relation (2) into Eq. (1), after simple transformation we get the following form of Eq. (1):

$$C\,\rho\frac{\partial T(x,t)}{\partial t} + v_x\frac{\partial T(x,t)}{\partial x} = \lambda\frac{\partial^2 T(x,t)}{\partial x^2}, \tag{3}$$

for $(x,t) \in \Omega$, where

$$C = c - L\frac{\partial f_s}{\partial t}$$

is called as the substantial thermal capacity. The above equation describes the heat conduction in the full homogeneous region (in the solid phase, in the two-phase zone and in the liquid phase).

Since function f_s, describing the volumetric solid state fraction, must satisfy the equalities

$$f_s(T_L) = 0 \quad \text{and} \quad f_s(T_S) = 1,$$

where T_L and T_S denote the liquidus and solidus temperatures,

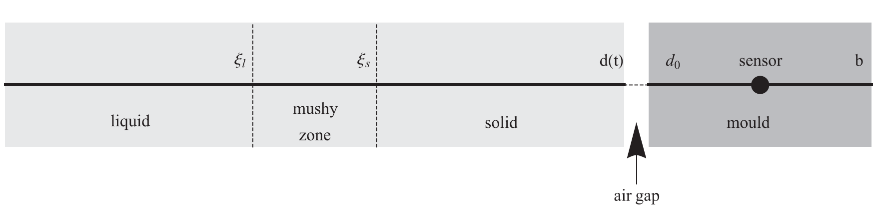

Fig. 1. Region of the problem.