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Estimation of river current using reduced Kalman filter finite element method

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ABSTRACT

The purpose of this paper is to investigate estimation of velocity, water elevation and contaminant concentration in a river current using the Kalman filter finite element method (KE-FEM), which is an improvement of the Kalman filter finite element method previously presented by the authors group. The state variables, velocity, water elevation and contaminant concentration, in the whole domain can be obtained by the finite element method. Combining the Kalman filter and the finite element method, a practically useful method can be obtained. The Kalman filter is an estimation method based on the system and observation equations. The non-liner shallow water equation is utilized for the state equation. The explicit Euler method is used for the temporal discretization. The finite element method based on the linear interpolation is employed for the spatial discretization. The long computational time is required for the computation by the previous method. To reduce the computational time, the computational domain is divided into two parts, the main and subsidiary domains. In the main domain, filtering procedures are carried out, whereas only a deterministic process is taken for the variables in the subsidiary domain. Eliminating the state variables in the subsidiary domain, the drastically efficient computation is carried out. The flow in the Teganuma river in Japan is computed as practical applications. Close agreement between observed and computed results was obtained.

1. Introduction

Recently, there are a lot of social demands for assessment on the occasion of new constructions especially in water areas, such as river, lake, shallow sea, etc. To satisfy such demands, numerical methods are one of the promising techniques. Careful observations at monitoring points in the area are also required. However, sometimes the observation is not satisfactory to know the physical phenomena in the whole area because the number of observation points is limited and observation errors are included in the data. To complement lack of the number of observations, numerical computations are one of the powerful tools for practical works.

Numerical results computed sometimes do not agree with measurements. There are sometimes unsolvable discussions between computationalists and observationalists whether the data measured are reliable or un-realistic to explain the physical phenomena. The observations include an amount of observation errors. Thus, the computation considering the system and observation errors is needed for the practical use. To exclude those errors from the computation considering the measurement data, the Kalman filter is one of the well established techniques.

To compensate lack of the observation and to exclude the observation and system errors, the Kalman filter finite element method has been originated and applied to the practical problems [4,12,8,11,2,14,10,13,7]. The Kalman filter is a useful tool to estimate sequential average values and corresponding covariances. The advantage of a combination between the Kalman filter and the finite element method is useful for the computation in both time and spatial directions. There are few papers published including flow models in the Kalman filter. The idea is also used in the field of meteorology and oceanology (e.g. [1,3,9,6,5]). Because a lot of observation data are collected in those fields, the ensemble methods are the present target of the research (e.g. [15]).

To apply the Kalman filter finite element method to practical use, the most annoying problem is to take long computational time especially for the matrix multiplication. To reduce the computational time, the reduced Kalman filter finite element method is presented in this paper. The whole computational domain can be classified into two domains, i.e. one is the main domain, in which the computation is mainly carried out and the other is the subsidiary domain, in which the auxiliary computation by the finite element method is performed. The state variables at the nodal points in the subsidiary domain are assumed un-correlated with those in the main domain. Those variables in the subsidiary domain is used only for the computation by the finite element method. The state variables in the subsidiary domain are used for the deterministic process. The Kalman filter including deterministic variables can easily be derived. The idea is applied to the estimation of velocity, water elevation and contaminant concentration in the Teganuma

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river located in Japan. A flow of river can be modeled by the linear shallow water equations. For the index of contaminant concentration, the dissolved oxygen, DO, is used because this index is very much related to habitation of living things in the water. To express the concentration of DO, the diffusion dispersion equation is employed. There are four observation points in the computed area of the Teganuma river, and at each point two horizontal components of velocity, water elevation and contaminant concentration are measured by the Ministry of Land, Infrastructure and Transport. Therefore, a total of 16 components of time series data are obtained. Based on the measurement data at 3 points, i.e. 12 components, the computation has been conducted. The obtained data at one point, i.e. 4 components, are used for the comparison between the computed and observed results. Substantial agreement between the computed and observed results was obtained. Thus, the adaptability of the reduced Kalman filter finite element method has been shown. The present method is practically useful because the computational time can be drastically reduced.

2. The Kalman filter

Natural phenomena can usually be expressed by the following equation:

$$x_{k+1} = D_k x_k + G_k w_k, \tag{1}$$

where Eq. (1) is referred to as the system equation, in which state variable at time t_k is denoted by x_k, D_k and G_k are transition and driving matrices, and w_k is system noise included on the occasion that the system is modeled and discretized, respectively. The observation y_k can not be obtained at a whole domain, but, at some limited observation points, thus

$$y_k = Hx_k + v_k, \tag{2}$$

where H means the coefficient matrix which expresses the correspondence between the observation and state variable and v_k is the observation noise,respectively. Both w_k and v_k are assumed as

$$w_k \sim N(0, Q), \tag{3}$$
$$v_k \sim N(0, R), \tag{4}$$

where $N(a, A)$ represents the normal distribution with mean and covariance A. It is also assumed that

$$E\{w_k, v_j\} = 0, \tag{5}$$

where $E\{\}$ is an expectation operator. Observation data are obtained at time t_k as

$$Y_k = [y_1, y_2, \ldots, y_k]. \tag{6}$$

The optimal estimate $\hat{x}_k$ is an expectation of x_k, giving the observation data Y_k,

$$\hat{x}_k = E\{x_k | Y_k\}. \tag{7}$$

The initial condition is given

$$\hat{x}_0 = \hat{v}_0, \tag{8}$$

where $\hat{v}_0$ is a specified value at the initial time t_0. The state estimator $\hat{x}_k$ using the new measurement y_k is of the form

$$\hat{x}_k = x_k^* + K_k[y_k - Hx_k^*], \tag{9}$$

where K_k is referred to as the Kalman gain, which will be given later in Eq. (16), and x_k^* is a priori estimate,

$$x_k^* = E\{x_k | Y_{k-1}\}. \tag{10}$$

The covariance P_k is derived as

$$P_k = E\{(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T\} = (I - K_k H)\Gamma_K, \tag{11}$$

where covariance Γ_k is defined as

$$\Gamma_k = E\{(x_k - x_k^*)(x_k - x_k^*)^T\} \tag{12}$$

and initial condition is

$$\Gamma_0 = \widehat{\Gamma}_0, \tag{13}$$

where $\widehat{\Gamma}_0$ is a specified value at the initial time t_0. One step ahead prediction of the state variable is obtained as

$$\hat{x}_{k+1} = D_k \hat{x}_k \tag{14}$$

then, it is found that

$$P_{k+1} = D_k \Gamma_k D_k^T + Q. \tag{15}$$

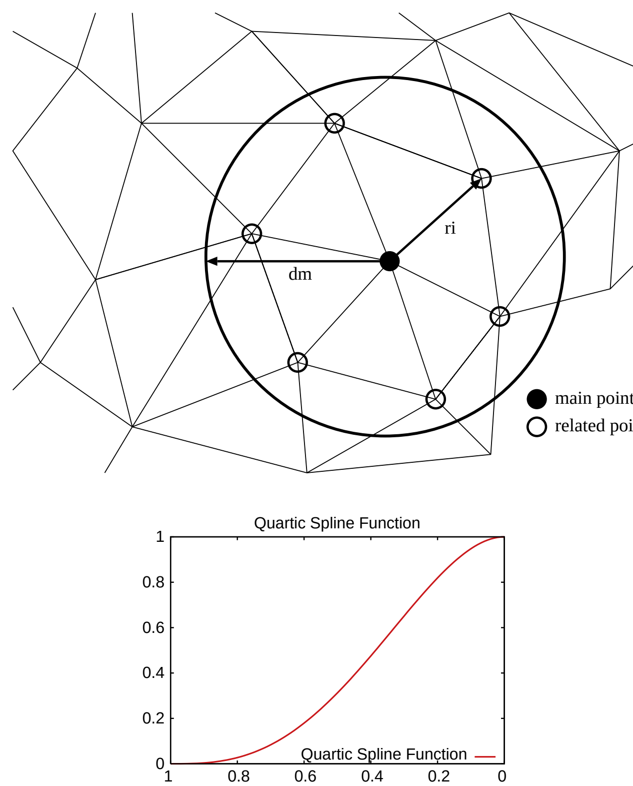

Fig. 1. Main point and related points in the main domain and quartic spline function.

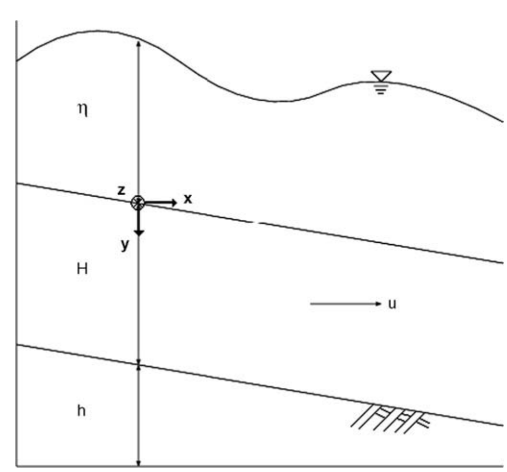

Fig. 2. Coordinate system.