International Communications in Heat and Mass Transfer 86 (2017) 1–11

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

A sharp-interface level-set method for compressible bubble growth with phase change

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ARTICLE INFO

Keywords:
Bubble growth
Compressible flow
Level-set method
Phase change

ABSTRACT

A sharp-interface level-set method is presented for simulating the growth and collapse of a compressible vapor bubble. The interface tracking method is extended to include the effects of bubble compressibility and liquid-vapor phase change by incorporating the ghost fluid method to efficiently implement the matching conditions of velocity, stress and temperature at the interface. The numerical results for one-dimensional compressible flows and spherical bubble growth show good agreement with the exact solutions. The level-set method is applied to investigate the effects of phase change, ambient temperature and wall on the compressible bubble growth and collapse.

1. Introduction

The growth and collapse of a compressible vapor bubble is of considerable interest in various applications such as thermal inkjet printing, hydrodynamic erosion, ultrasonic cleaning and medical treatment. However, its general predictive model including the effects of bubble compressibility and liquid-vapor phase change is lacking in the literature.

Extensive numerical works were conducted for the compressible bubble motion using interface tracking methods such as a level-set (LS) method [1–4] a front-tracking method [5] and the volume-of-fluid (VOF) method [6]. The interface tracking methods were coupled to the ghost fluid method (GFM) [1], which is a numerical technique for efficiently implementing the matching conditions at the interface. However, the numerical methods were limited to the bubble motion without phase change. Computational efforts were also made for bubble growth with phase change using a moving-grid method [7,8], the front-tracking method [9,10], the LS method [11–13], and the VOF method [14,15]. However, only a few numerical studies were extended to the compressible bubble growth and collapse with phase change.

Houim and Kuo [16] developed the LS method for compressible flows with phase change as well as chemical reaction. They solved quite complex flows of shock wave and droplet evaporation with chemical reaction considering the velocity, stress, temperature and heat flux jump conditions at the vaporizing interface. However, the temperature condition at the interface related to the saturation temperature, which is essentially important in the phase change problems, was not clearly implemented in their method.

In the earlier work of Suh and Son [17], the LS method was applied to the bubble growth and collapse in a thermal inket process. The LS method was modified to include the effect of phase change at the liquid-vapor interface. The compressibility effect of a bubble was also included to account for the high vapor pressure caused by homogeneous bubble nucleation. However, the method has numerical limitations by assuming that the vapor density is uniform inside the bubble and using the diffuse-interface formulation based on a smoothed delta function. The formulation with being smeared out over several grid spacings has difficulties in accurately imposing the interface temperature condition and the velocity jump condition due to phase change.

In this work, the LS method is improved to include the effects of bubble compressibility and liquid-vapor phase change by incorporating the GFM [1,18-22] to sharply enforce the matching conditions at the interface. The numerical method is validated through computations of one-dimensional compressible flows, whose exact solutions are available, and is applied to investigate the effect of phase change, ambient temperature and wall on the compressible bubble growth and collapse.

2. Mathematical formulation

The present numerical approach is based on the sharp-interface LS formulation developed in our previous studies [23–26] for incompressible flows with phase change. The LS method is extended for the compressible vapor bubble motion with phase change. The liquid-vapor interface is tracked by the LS function ϕ, which is defined as a signed distance from the interface. The positive sign is chosen for the liquid phase and the negative sign for the vapor phase. In this work, we

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http://dx.doi.org/10.1016/j.icheatmasstransfer.2017.05.016

Nomenclature

a	= speed of sound
c	= specific heat at constant volume
E	= specific total energy
F	= fraction function
h	= grid spacing
h_{lv}	= latent heat of vaporization
$\dot{m}$	= mass flux across the interface
$\mathbf{n}$	= unit normal vector
p	= pressure
r, y	= cylindrical coordinates
R	= bubble radius
R_E	= bubble radius at the mechanical equilibrium
R_T	= bubble radius at the thermal equilibrium
t	= time
t^*	= artificial time
$\mathbf{u}$	= flow velocity vector, (u,v)
$\mathbf{U}$	= interface velocity vector
x, y	= Cartesian coordinates

Greek symbols

α	= step function
β	= $\rho_v^{-1} - \rho_l^{-1}$
γ	= specific heat ratio
κ	= interface curvature
λ	= thermal conductivity
μ	= dynamic viscosity
ρ	= density
σ	= surface tension coefficient
τ	= viscous stress
ϕ	= distance function from the liquid-vapor interface
Φ	= viscous dissipation function

Subscripts

f	= fluid
I	= interface
l, v	= liquid, vapor
o	= initial
sat	= saturation
wc	= wall in the center region
∞	= ambient

Superscripts

G, R	= ghost, real
n	= time step
^	= effective property
*	= intermediate step

assume that (1) the flow is laminar; (2) the vapor phase is compressible whereas the liquid phase is incompressible; (3) all fluid properties except vapor density are constant in each phase; (4) the vapor phase is at the saturation temperature while considering phase change problems.

The conservation equations of mass, momentum and energy for each phase are written as

$$\frac{\partial \rho_f}{\partial t} + \nabla\cdot\mathbf{u}_f\rho_f = 0 \tag{1}$$

$$\frac{\partial \rho_f \mathbf{u}_f}{\partial t} + \nabla\cdot\mathbf{u}_f\rho_f\mathbf{u}_f = -\nabla p_f + \nabla\cdot\tau_f \tag{2}$$

$$\frac{\partial \rho_f E_f}{\partial t} + \nabla\cdot\mathbf{u}_f\rho_f E_f = -\nabla\cdot p_f\mathbf{u}_f + \nabla\cdot\lambda_f\nabla T_f + \nabla\cdot\tau_f\cdot\mathbf{u}_f \tag{3}$$

where the subscript f denotes the liquid phase (l) for $\phi > 0$ and the vapor phase (v) for $\phi \le 0$ and the viscous stress τ_f and the specific total energy E_f are expressed as

$$\tau_f = \mu_f\left[\nabla\mathbf{u}+\nabla\mathbf{u}^T-\frac{2}{3}(\nabla\cdot\mathbf{u})\mathbf{I}\right]_f \tag{4}$$

$$E_f = c_f T_f + \frac{1}{2}\mathbf{u}_f\cdot\mathbf{u}_f \tag{5}$$

The conservation equations for each phase are coupled through the matching conditions at the interface ($\phi = 0$):

$$\mathbf{u}_l - \mathbf{u}_v = \beta\dot{m}\mathbf{n} \tag{6}$$

$$\mathbf{n}\cdot[(p_v-p_l)\mathbf{I}+\tau_l-\tau_v] = (\sigma\kappa - \beta\dot{m}^2)\mathbf{n} \tag{7}$$

$$T_l = T_v = T_I = T_{sat}(p_v) \tag{8}$$

where $\beta = \rho_v^{-1} - \rho_l^{-1}$. The interface normal $\mathbf{n}$, the interface curvature κ, and the mass flux $\dot{m}$ across the interface due to phase change are defined as

$$\mathbf{n} = \nabla\phi/|\nabla\phi| \tag{9}$$

$$\kappa = \nabla\cdot\mathbf{n} \tag{10}$$

$$\dot{m} = \rho_f(\mathbf{U} - \mathbf{u}_f)\cdot\mathbf{n} \tag{11}$$

where $\mathbf{U}$ is the interface velocity. The mass flux $\dot{m}$ is determined from the following energy balances at the interface

$$\dot{m} = \frac{1}{h_{lv}}\mathbf{n}\cdot\lambda_l(\nabla T)_l \tag{12}$$

The LS function ϕ is advanced and reinitialized as

$$\frac{\partial\phi}{\partial t} + \mathbf{U}\cdot\nabla\phi = 0 \tag{13}$$

$$\frac{\partial\phi}{\partial t^*} = \frac{\phi}{\sqrt{\phi^2+h^2}}(1 - |\nabla\phi|) \qquad \text{if} \quad |\phi| \ge h/2 \tag{14}$$

where h is a grid spacing, t^* is an artificial time for iterative calculation and the interface velocity is evaluated using the liquid velocity as

$$\mathbf{U} = \mathbf{u}_l + \frac{\dot{m}\mathbf{n}}{\rho_l} \tag{15}$$

3. Numerical approach

The governing equations are discretized in a staggered grid system where the velocity components (u,v) are defined at cell faces whereas the other dependent variables are defined at cell centers. A second-order, essentially nonoscillatory (ENO) scheme is used for the advection terms and the distance function, and a second-order central difference scheme for the other terms. While discretizing the governing equations temporally, we use a first-order explicit scheme for the advection and source terms and an implicit scheme for the diffusion terms.

The LS advection equation is discretized as

$$\frac{\phi^{n+1}-\phi^n}{\Delta t} + (\mathbf{U}\cdot\nabla\phi)^n = 0 \tag{16}$$