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Mixed convection of MHD flow in nanofluid filled and partially heated wavy walled lid-driven enclosure

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ABSTRACT

A computational work has been done to investigate the effects of mixed convection of MHD flow in nanofluid filled and partially heated wavy walled lid-driven enclosure. Finite difference method is used to solve governing equations of mixed convection for different parameters as Hartmann number, Richardson number, nanoparticle volume rate in partially heated and wavy walled enclosure. It is found that the rate of heat transfer decreases with increasing the Hartmann number. The rate of heat transfer can be enhanced or reduced by increasing the volume fraction of nanoparticles based on Hartmann and Richardson numbers.

1. Introduction

Mixed convection heat transfer and fluid flow in complex shaped geometry are important in engineering due to its wide applications, such as cooling of electronical devices, heat exchangers, solar collectors, cooling or heating of buildings. Besides, partial heating of confined spaces attracted many researchers recently. Partial heater was applied in some papers in open literature as seen in review of Oztop et al. [1]. There are many example on partial cooling or heating applications as Guimaraes and Menon [2]. Pioneer of computational studies of natural convection in a nanofluid filled enclosures is Khanafer et al. [3]. Oztop and Abu-Nada [4] conducted a computational solution on natural convection in enclosures with Cu-water nanofluid. They found that heat transfer increases with increasing of length of partial heater and nanoparticle volume fraction. Also, effects of combined convection (natural + forced convection) are studied in nanofluid filled systems. In this context, Mehrez et al. [5] focused on entropy generation analysis in the assisting flow of Cu-water nanofluid in an inclined open cavity and they observed that the main important parameter is the inclination angle. Hussain et al. [6] investigated the effects of magnetic field on entropy generation due to mixed convection of water-alumina nanofluid flow in a double lid driven cavity with discrete heating.

In some problems in engineering, the geometry can be curvilinear. Sheremet et al. [7] solved a problem computationally to make analysis of MHD free convection in a wavy open porous tall cavity filled with nanofluids. The cavity has a corner heater. They obtained that heat transfer enhancement with Rayleigh number (*Ra*) and heat transfer reduction with Hartmann (*Ha*) number, while magnetic field inclination angle leads to non-monotonic changes of the heat transfer. Heated wavy walled cavity was applied to a melting problem by Kousksou et al. [8]. In their case, the bottom wavy wall is heated isothermally and the problem solved via finite volume method. Waviness of the wall plays important role on the temperature distribution and flow field and they observed that the rate of the melting increases with the elevation in the magnitude of the amplitude value of the wavy surface. As an original work on insulated wavy walled cavity with natural convection of Al_2O_3/water nanofluids is studied by Abu-Nada and Oztop [9]. Influence of a magnetic field on the natural convection and entropy generation was studied by Cho [10] for Cu–water nanofluid in a closed space with complex-wavy surfaces. Their obtained results showed that the mean Nusselt number decreases and entropy generation increases with an increasing wave amplitude. Abu-Nada and Chamkha [11] solved a problem of mixed convection in a water-CuO filled lid-driven cavity with wholly heated wavy wall. They observed that heat transfer

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Nomenclature

B	geometry parameter
B_o	magnetic field strength
c_P	specific heat at constant pressure (kJ kg^{-1} K^{-1})
g	gravitational acceleration (m s^{-2})
H	height and width of the cavity (m)
Ha	Hartmann number
h	local heat transfer coefficient (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
Nu	Nusselt number
Pr	Prandtl number
q_w	heat flux (W m^{-2})
Ra	Rayleigh number
Re	Reynolds number
Ri	Richardson number
T	temperature (K)
u, v	dimensional x and y components of velocity (m s^{-1})
U, V	dimensionless x and y components of velocity
$x^{\prime}, y^{\prime}$	dimensional coordinates (m)
x, y	dimensionless coordinates

Greek symbols

α	fluid thermal diffusivity (m^2 s^{-1})
β	thermal expansion coefficient (K^{-1})
ε	numerical tolerance
φ	Nanoparticle volume fraction
ν	kinematic viscosity (m^2 s^{-1})
θ	dimensionless temperature
ψ	dimensional stream function (m^2 s^{-1})
Ψ	dimensionless stream function ∞ ∞ φσ
ω	dimensional vorticity (s^{-1})
Ω	dimensionless vorticity
ρ	density (kg m^{-3})
σ	electrical conductivity (Ω^{-1} m^{-1})
φ	volume fraction of nanoparticles, (dimensionless)
μ	dynamic viscosity (N s m^{-2})

Subscripts

avg	average
C	cold
cp	centi poise
f	fluid
H	hot
nf	nanofluid
p	particle
w	wall

enhancement with Rayleigh number and heat transfer reduction with Hartmann number, while magnetic field inclination angle leads to non-monotonic changes of the heat transfer rate. Bondereva et al. [12] solved a problem numerically to simulate the heat transfer in a wavy walled and nanofluid filled cavity with a corner heater. They found that an increase in the *Ha* number leads to an attenuation of the convective flow and heat transfer reduction. Study on laminar mixed convection was performed by Akbarinia et al. [13] to see the effects of nanofluid in horizontal curved tubes. Other related works can be found in refs. Sheikholeslami et al. [14], Sathiyamoorthy and. Chamkha [15] and Kefayati et al. [16].

The main aim of this work is to study the effects of magnetic field on mixed convection heat transfer and fluid flow in a wavy walled cavity with lid-driven wall filled with nanofluid. The originality of this work is the heated wavy bottom wall.

2. Definition of considered physical model

The considered physical model is depicted in Fig. 1. In this model, the wavy wall is heated partially. The gravity acts in − y direction and magnetic field acts parallel to x-axis. The height of the hill is given by B and length of it is L which are governing parameters on heat and fluid flow. The ceiling of the cavity moves from left to right with constant velocity. Geometrically, the length and width of the cavity, which are given by W and H, are equal. The vertical walls are considered adiabatic.

3. Governing equations and problem formulation

Fig. 1 shows a schematic diagram of the wavy walled differentially heated enclosure in a presence of magnetic field. The nanofluid is assumed incompressible and the flow is assumed to be laminar. It is assumed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. Thermo-physical properties of the nanofluid are assumed to be constant except for the density variation, which is approximated by the Boussinesq model. The governing equations for the laminar, two-dimensional, steady state natural convection in the presence of magnetic field in terms of the stream function-vorticity formulation are written as

Vorticity

$$\frac{\partial}{\partial x'}\left(\omega\frac{\partial\psi}{\partial y'}\right)-\frac{\partial}{\partial y'}\left(\omega\frac{\partial\psi}{\partial x'}\right)=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial\omega}{\partial x'^2}+\frac{\partial\omega}{\partial y'^2}\right)+\beta_{nf}g\left(\frac{\partial T}{\partial x'}\right)+\frac{\sigma_{nf}B_o^2}{\rho_{nf}}\frac{\partial^2\psi}{\partial x^2} \tag{1}$$

Energy

$$\frac{\partial}{\partial x'}\left(T\frac{\partial\psi}{\partial y'}\right)-\frac{\partial}{\partial y'}\left(T\frac{\partial\psi}{\partial x'}\right)=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x'^2}+\frac{\partial^2 T}{\partial y'^2}\right) \tag{2}$$

Kinematics

$$\frac{\partial^2\psi}{\partial x'^2}+\frac{\partial^2\psi}{\partial y'^2}=-\omega \tag{3}$$