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Transition to asymmetry of transitional round fountains in a linearly stratified fluid

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ABSTRACT

In this study, a series of three-dimensional direct numerical simulations were carried out for transitional round fountains in a linearly-stratified fluid over the ranges of $100 \leq Re \leq 400$, $1 \leq Fr \leq 8$ and $0.0 \leq s \leq 0.3$, where Fr, Re and s are the Froude, Reynolds, and dimensionless temperature stratification parameters, respectively, to examine, both qualitatively and quantitatively, the effect of these parameters on their transition to asymmetry and the asymmetric behavior. It is found that the transition to asymmetry are well represented and quantified by azimuthal velocity, with non-zero or noticeable azimuthal velocity indicating asymmetry. The results show that when Fr or Re are small, a fountain remains axisymmetric for all time; however, when Fr or Re are increased to be sufficiently large, the fountain will be axisymmetric initially, before becoming asymmetric and unsteady, ultimately reaching a fully developed quasi-steady stage when each quantity fluctuates over a constant, time-average, value. The stratification is found to play a positive role to stabilize the flow and to reduce or even to eliminate the asymmetric behavior. The numerical results were also used to develop the scaling for the time for transition to asymmetry, which is found to be strongly dependent on Fr and s, while only weakly dependent on Re.

1. Introduction

Fountains, which are negatively buoyant jets, have been, and will continue to be, a topic of strong research interest since the 1950s due to their fundamental and application significance (see, *e.g.* [1] for a recent review on the topic). A fountain is produced by injected a heavier fluid upward into a lighter fluid or vice versa. The resultant buoyancy opposes the momentum of the jet flow, leading to gradually reduced vertical jet velocity until it becomes zero at a certain finite height, followed by the jet flow changing its direction and flowing back around the core of the upward/downward flow, with an intrusion forming on the bottom which moves outwards.

If a fountain is injected into a linearly stratified ambient fluid its behavior will be dictated by the Reynolds Number Re, the Froude Number Fr, and the stratification parameter S_p, which are defined as follows,

$$Re = \frac{W_0 X_0}{\nu}, \quad Fr = \frac{W_0}{\sqrt{gX_0(\rho_0-\rho_a)/\rho_0}} = \frac{W_0}{\sqrt{gX_0\beta(T_a-T_0)}}, \quad S_p = -\frac{1}{\rho_{a,0}}\frac{d\rho_{a,Z}}{dZ}, \tag{1}$$

where X_0 is the radius of the orifice at the fountain source, W_0 is the mean inlet velocity of the jet fluid at the source, g is the acceleration due to gravity, ρ_0, T_0 and ρ_a, T_a are the densities and temperatures of the jet fluid and the ambient fluid at the source, ν and β are the kinematic viscosity and the coefficient of volumetric expansion of fluid, and $\rho_{a,0}$ and $\rho_{a,Z}$ are the initial densities of the ambient fluid at the bottom (at $Z = 0$) and at height Z, with Z being the coordinate in the vertical direction. The second expression of Fr in the above equation applies when the density difference is due to the difference in temperature of the jet and ambient fluids using the Oberbeck-Boussinesq approximation.

If the Oberbeck-Boussinesq approximation is valid, S_p can be represented by the following temperature stratification parameter, S,

$$S = \frac{dT_{a,Z}}{dZ} = \frac{S_p}{\beta}, \tag{2}$$

where $T_{a,Z}$ is the initial temperature of the ambient fluid at the height Z. However, the dimensionless form of S, as defined as follows, is usually used instead,

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$$s = \frac{d\theta_{a,z}}{dz} = \frac{X_0}{(T_{a,0}-T_0)}S = \frac{X_0}{\beta\,(T_{a,0}-T_0)}S_p, \tag{3}$$

where $\theta_{a,z} = (T_{a,Z} - T_{a,0})/(T_{a,0} - T_0)$ and $z = Z/X_0$ are the dimensionless initial temperature of the ambient fluid at Z and the dimensionless height, respectively, and $T_{a,0}$ is the initial temperature of the ambient fluid at the bottom.

The previous studies have focused on single fountains in a homogeneous fluid (*i.e.*, $s = 0$), which is either 'very weak' when $Fr \le 1$, or 'weak' when $1 \le Fr \le 3$, or 'forced' when $Fr \ge 3$, as classified by Kaye and Hunt [2] and Burridge and Hunt [3]. The behavior of a forced fountain is found to be quite different from that of a weak or very weak round fountain, as demonstrated in, *e.g.* [1–14].

The most common parameter used in characterizing fountain behavior is the maximum fountain penetration height Z_m and numerical scaling and empirical relations have been developed to quantify it over a wide range of Re, Fr, s under different configurations. The readers are referred to, *e.g.*, [1,3-5,11,12,15-25], for some studies on the this characteristics of fountains.

The transition to asymmetry in fountains is key to elucidate the mechanism for the onset of entrainment in fountains, but the understanding is current scarce. There have been some recent studies on transitional fountains, aiming at providing some insights into the issue. Lin and Armfield [11] studied the onset of entrainment in transitional round fountains in a homogeneous fluid over the ranges of $1 \le Fr \le 8$ and $200 \le Re \le 800$ using DNS, and found that entrainment is strongly dependent on Re but the effect from Fr is relatively much smaller. Williamson et al. [14] investigated the transitional behavior of weak turbulent round fountains in a homogeneous fluid over a wide range of Re (20 to 3494), although Fr was relatively small, at $0.1 \le Fr \le 2.1$, and observed that there is a continuum of behavior over this transitional Fr range, from hydraulically driven buoyancy dominated flow to momentum dominated flow. However, no study has been found in which the onset of asymmetry in transitional round fountains in stratified fluids has been investigated, which motivates the current study.

In this study, a series of three-dimensional DNS runs were carried out for transitional round fountains in linearly stratified fluids over the ranges of $100 \le Re \le 400$, $1 \le Fr \le 8$ and $0.0 \le s \le 0.3$ to examine the effect of Fr, Re and s on the onset of asymmetry and unsteadiness of these transitional round fountains.

The remainder of this paper is organized as follows. The physical system under consideration, the governing equations and the initial and boundary conditions, and the numerical methods for DNS are briefly described in Section 2. The transition to asymmetry and the asymmetric behavior of the transitional round fountains over the ranges of $100 \le Re \le 400$, $1 \le Fr \le 8$ and $0.0 \le s \le 0.3$ are described and discussed in Section 3, both qualitatively and quantitatively, with the DNS results. The conclusions are drawn in Section 4.

2. Numerical method

The physical system under consideration is a vertical circular container of the dimensions $H \times R$ (Height $\times$Radius), containing a Newtonian fluid initially at rest and at either a uniform temperature of T_a (in the homogeneous case) or a constant temperature gradient $dT_{a,Z}/dZ$ (in the stratified case), as sketched in Fig. 1 of [26], where the origin of the Cartesian coordinate system (X, Y, Z) is at the center of the bottom. The vertical sidewall is non-slip and insulated and the top (at $Z = H$) is open. On the bottom center (at $Z = 0$), an orifice with radius X_0 is used as the fountain discharge source. The remaining bottom region is a rigid non-slip and insulated boundary. At time $t = 0$, a stream of fluid at T_0 ($T_0 < T_a$ for the homogeneous case or $T_0 < T_{a,0}$ for the stratified case) is injected upward into the container from the source to initiate the fountain flow and this discharge is maintained over the whole course of a specific DNS run.

The flow is governed by the three-dimensional incompressible Navier-Stokes and temperature equations, with the Oberbeck-Boussinesq approximation for buoyancy, which are written in conservative form in Cartesian coordinates as follows,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0, \tag{4}$$

$$\frac{\partial U}{\partial t} + \frac{\partial (UU)}{\partial X} + \frac{\partial (VU)}{\partial Y} + \frac{\partial (WU)}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \nu\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}+\frac{\partial^2 U}{\partial Z^2}\right), \tag{5}$$

$$\frac{\partial V}{\partial t} + \frac{\partial (UV)}{\partial X} + \frac{\partial (VV)}{\partial Y} + \frac{\partial (WV)}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \nu\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}+\frac{\partial^2 V}{\partial Z^2}\right), \tag{6}$$

$$\frac{\partial W}{\partial t} + \frac{\partial (UW)}{\partial X} + \frac{\partial (VW)}{\partial Y} + \frac{\partial (WW)}{\partial Z} = -\frac{1}{\rho}\frac{\partial P}{\partial Z} + \nu\left(\frac{\partial^2 W}{\partial X^2}+\frac{\partial^2 W}{\partial Y^2}+\frac{\partial^2 W}{\partial Z^2}\right) + g\beta(T - T_{a,Z}), \tag{7}$$

$$\frac{\partial T}{\partial t} + \frac{\partial (UT)}{\partial X} + \frac{\partial (VT)}{\partial Y} + \frac{\partial (WT)}{\partial Z} = \kappa\left(\frac{\partial^2 T}{\partial X^2}+\frac{\partial^2 T}{\partial Y^2}+\frac{\partial^2 T}{\partial Z^2}\right), \tag{8}$$

where U, V, and W are the velocity components in the X, Y, and Z directions, t is time, P is pressure, T is temperature, and ρ, ν, and κ are the density, kinematic viscosity, and thermal diffusivity of fluid, respectively. The gravity is acting in the negative Z (vertical) direction.

The appropriate initial conditions are $U = V = W = 0$, $T(Z) = T_{a,0} + s(T_{a,0} - T_0)Z/X_0$ everywhere in the computational domain when $t \le 0$. The appropriate boundary conditions for $t > 0$ are $U = V = W = 0$ and the first gradient of temperature with respect to the horizontal direction is zero on the vertical sidewall; $U = V = W = 0$ and $\partial T/\partial Z = 0$ on the bottom floor except at the fountain source where $U = V = 0$, $W = W_0$ and $T = T_0$; and $\partial U/\partial Z = \partial V/\partial Z = \partial W/\partial Z = \partial T/\partial Z = 0$ on the top boundary of the domain, respectively.

The above governing equations were discretized on a non-uniform mesh using a finite volume method, with a standard 2nd-order central difference scheme used for the viscous and divergence terms and the 3rd-order QUICK scheme for the advection terms. The 2nd-order Adams-Bashforth and Crank-Nicolson schemes were used for the time integration of the advective and diffusive terms, respectively. The PRESTO (PREssure STaggering Option) scheme was used for the pressure gradient. The ICEM technique was used to create O-Type Multiblock Hexahedron meshes. To ensure accuracy of the DNS runs, non-uniform meshes with finer grids in regions of larger parameter gradients were used. All DNS were carried out using ANSYS Fluent 13.

There were totally 51 DNS runs carried out in this study, with the key information about these runs listed in Table 1, which was also listed in Table 1 of [26]. However, in the current paper the occurrence of asymmetric behavior, which will be described and discussed in detail in Section 3, is also noted for each run. The DNS runs mainly focus on Fr and Re over the ranges $1 \le Fr \le 8$ and $100 \le Re \le 400$ with $s = 0.03$, 0.09 and 0.18, which represent a very weak, a weak and an average stratification. Additionally, DNS runs with s over a wider range of $0.03 \le s \le 0.3$ were carried out for the specific case of $Fr = 2$ and $Re = 200$, aiming to further examine the effect of the stratification. Furthermore, DNS runs with $s = 0$, which corresponds to a homogeneous ambient, were also carried out for all Fr values and some of the Re values considered, for the purpose of comparison. In all DNS runs, $H = 0.3$ m and $R = 0.6$ m were chosen, which were found to be sufficiently large and appropriate for the problem addressed here, based on our experience and testing. $T_{a,0}$ was fixed at 300 K and the time step was fixed at 0.05 s. Extensive mesh-dependence and time-step-dependence testing was carried out to ensure that the chosen meshes and time-steps for the DNS runs provide accurate numerical results. The meshes used for the DNS runs have grids ranging from 3.13 million to 4.36 million cells, as shown in Table 1.

Although all parameter values obtained from the DNS runs are