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Heat and mass transfer within a vertical pipe with a surface heating element of variable size

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ABSTRACT

Heat and mass transfer due to upstream fluid flow in a vertical pipe which is heated in some region due to an external heating element on the surface of the pipe is considered. Unlike most studied in the literature which consider heating uniformly over the entire pipe, we allow for the heater to act over a smaller sub-region of the pipe surface. We first derive a heat and mass transfer model to describe the velocity, pressure, and temperature evolution in a vertical pipe under the assumption of cylindrical symmetry. Using a finite element method we are able to obtain numerical simulations to this model. We compare solutions under a variety of different heater configurations, in order to understand how the size and placement of the heating element on the surface of the pipe will modify the thermal properties of the fluid. We find that a smaller heating element placed near the top of the pipe can still deliver sufficient heat so that the temperature of fluid exiting the top of the pipe has desirable thermal properties for a specific application, and in such cases it is not necessary to heat the entire length of the pipe. Such a configuration could be more efficient, as it requires less material for the heating element, while also requiring less energy for the heating. On the other hand, if the heating element is too small, or poorly placed along the pipe, then it may not be possible to obtain desirable thermal properties in the fluid that would have been possible with a heating element covering the entire pipe length.

1. Introduction

The upward flow of a fluid in a duct or pipe which is heated has been the topic of many studies. Single phase flow is defined as regular flow of a single fluid of a single phase along some domain. Regular flow in a pipe has been studied by Avila et al. [1] to establish when transition to turbulent flow occurs. The effects of a heating element applied in pipe flow with a part of the pipe being insulated have been studied by Faghri et al. [2] through numerical simulations. A similar case where the heating was applied by a step change was studied by Bilir [3], again through numerical simulations. Bernier and Baliga [4] performed a numerical investigation of conjugate conduction and laminar mixed convection in vertical pipes for upward flow and uniform wall heat flux. One application of such numerical studies would be to boreholes in vertical ground heat exchangers [5]. Turbulence under $k-\epsilon$ models was numerically studied by Hiroaki et al. [6] for the forced and natural convection heat transfer in an upward flow due to a uniformly heated vertical pipe.

There have been some analytical studies on the upward flow of a fluid in a uniformly heated pipe as well, often for the case where the flow is assumed one-dimensional. An exact solution for the problem of laminar convective flow under a pressure gradient along a vertical pipe in the case where the walls of the pipe are heated uniformly (under the assumption that velocity and buoyancy profiles far from the pipe entrance do not change with height, and entry-length effects are ignored) was given by Morton [7]. In that study, it was shown that when an upflow is heated the velocity near the walls is increased and that near the axis decreased until, for sufficiently large Rayleigh numbers, definite velocity and thermal boundary layers are formed. An exact solution was given by Gupta [8] for the problem of unsteady laminar convective flow under a pressure gradient along a vertical pipe which was heated uniformly. Barletta and Lazzari [9] analytically study fully developed mixed convection flow in a vertical circular duct assuming laminar parallel flow. In this study, the wall heat flux is assumed uniform in the axial direction yet depends on the angular variable, so that a proportion of the pipe can be viewed as heated and insulated. Smith et al. [10] analytically studied the temperature distribution in laminar pipe flow with a step change in wall heat flux.

Despite all of the previously mentioned interest in vertical pipe flow within a heated pipe, there has been little work on partially heated

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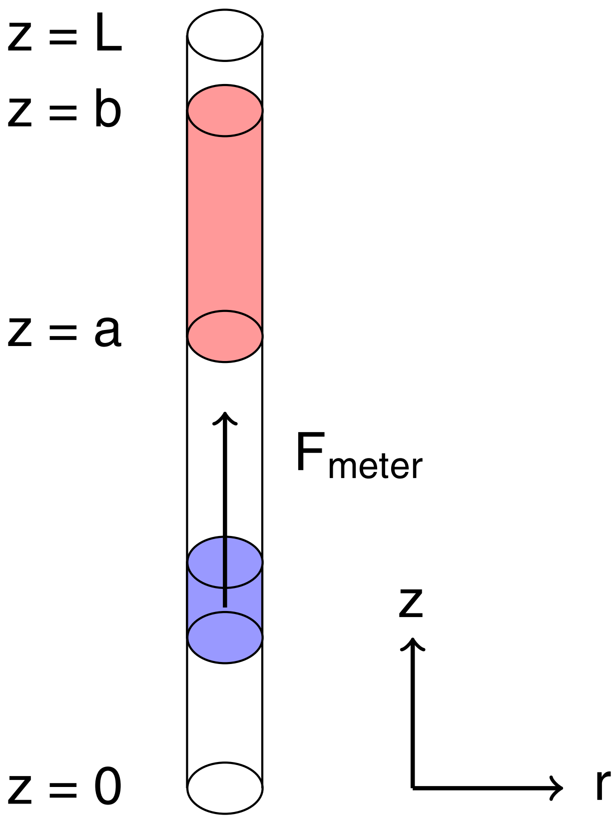

Fig. 1. Schematic representation of the system.

pipes, that is, pipes which are heated symmetrically only over a segment on the pipe. Applications might include coffee makers [11,12], nuclear reactors, refrigerator systems [13], and thermosyphons [14], for which it may be adventagious or more efficient to heat only a segment of the pipe rather than the entire length of the pipe. Motivated by this, in this paper we shall consider upward fluid flow within a pipe, in the case where only a cylindrical segment of the pipe is heated. The system of interest consists of a cylindrical pipe of length L that exerts some electrical push $\mathbf{F}_{meter}$ in the blue region and the red region ($a \leq z \leq b$) represents heat applied on the boundary, as in Fig. 1. Note that the meter may be calibrated to give pulses. Pulsating heat pipes have been studied before in some contexts, but again under the assumption of uniform heating. For instance, Charoensawan et al. [15] consider pulsating heat pipes with the entire apparatus submerged in a heat bath which provides the heating. In addition to the applications listed above, recent applications of heated pipe flow to nanofluids have also been considered [16–18].

For the development of our mathematical model, we choose to consider the case of single phase flow, corresponding to the case where a heater is applied in some region of the pipe, heating the fluid, but never reaching saturation temperature where evaporation begins. This is particularly useful in applications where a fluid is being heated but no boiling or transition to other phases will occur or are desired. In the case of a boiler (rather than a heater) placed on the pipe, there will be a phase transition, and a multiphase model will be needed. We consider this case in a companion paper [19], where we study a one-dimensional model for a boiler placed along a cylindrical subsection of a larger cylindrical pipe.

The paper is organized as follows. In Section 2 we derive a heat and mass transfer model under the assumption of cylindrical geometry, and then we provide a non-dimensionalisation of this model. In Section 3 we give the numerical solution approach (a finite element method), and we give numerical results and discuss them in Section 4, in order to deduce the influence of the heater on the vertical pipe flow. We give concluding remarks in Section 5.

2. Derivation of the heat and mass transfer model

We assume incompressible, Newton flow of density ρ, viscosity μ, and velocity field $\mathbf{u} = (u(r,\theta,z), v(r,\theta,z), w(r,\theta,z))$ in the system of Fig. 1. We assume that the temperature never passes the critical point where evaporation begins, and therefore no phase changes of the fluid occur.

To model the system we use the Boussinesq approximation [20] as in [21] and ignore any variations other than the density variation. The Boussinesq approximation is only valid when the difference in density $\Delta\rho$ is small compared to some constant density ρ_0, in this case, the density of the fluid at the initial temperature. This holds in our case, as we are modelling flow of water or similar fluids. Variations of the density are ignored, except for when they give rise to gravitational force. Therefore, the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0, \tag{1}$$

becomes

$$\nabla\cdot\mathbf{u} = 0. \tag{2}$$

In the momentum equation

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla p + \mu\nabla^2\mathbf{u} + \mathbf{F}, \tag{3}$$

we replace ρ by ρ_0. The term $\mathbf{F}$ represents the contribution of any forces that act on a volume element of fluid. In this case, we have the force from the meter $\mathbf{F}_{meter}$ and a force for the effect of gravity

$$\mathbf{F}_g = \rho\mathbf{g}, \tag{4}$$

where the gravitational acceleration is derivable from a potential

$$\mathbf{g} = -\nabla\Phi. \tag{5}$$

In this case, we expect the variations in density to be important and therefore let

$$\rho = \rho_0 + \Delta\rho. \tag{6}$$

Therefore,

$$\mathbf{F}_g = -(\rho_0 + \Delta\rho)\nabla\Phi, \tag{7}$$

which implies

$$\mathbf{F}_g = -\nabla(\rho_0\Phi) + \Delta\rho\mathbf{g}. \tag{8}$$

Letting $P = p + \rho_0\Phi$ and substituting into the momentum equation we obtain

$$\rho_0\frac{D\mathbf{u}}{Dt} = -\nabla P + \mu\nabla^2\mathbf{u} + \Delta\rho\mathbf{g} + \mathbf{F}_{meter}. \tag{9}$$

The reason why we can treat ρ as a constant except in the term involving gravitational forces is that the term $\Delta\rho\mathbf{g}$ can produce significant effects even when $\Delta\rho/\rho_0 \ll 1$ [21]. In this case, we linearize the dependence of ρ on the temperature T

$$\Delta\rho = -\alpha\rho_0\Delta T, \tag{10}$$

where α is the coefficient of expansion of the fluid. We therefore obtain the Boussinesq equation

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\mathbf{u} - \alpha\Delta T\mathbf{g} + \frac{1}{\rho}\mathbf{F}_{meter}, \tag{11}$$

where we have renamed ρ_0 into ρ and P into p, and $\nu = \mu/\rho$ is the kinematic viscosity. The term $-\alpha\Delta T\mathbf{g}$ represents the buoyancy force.

We now need a temperature equation. We further assume that the fluid has a constant heat capacity per unit volume, ρC_p. Then

$$\rho C_p\frac{DT}{Dt} = \text{rate of heating per unit volume of fluid}. \tag{12}$$