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Treatment of acoustic fluid–structure interaction by localized Lagrange multipliers and comparison to alternative interface-coupling methods

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ABSTRACT

This paper is a sequel on the topic of localized Lagrange multipliers (LLM) for applications of fluid–structure interaction (FSI) between finite-element models of an acoustic fluid and an elastic structure. The prequel paper formulated the spatial-discretization methods, the LLM interface treatment, the time-marching partitioned analysis procedures, and the application to 1D benchmark problems. Here, we expand on formulation aspects required for successful application to more realistic 2D and 3D problems. Additional topics include duality relations at the fluid–structure interface, partitioned vibration analysis, reduced-order modeling, handling of curved interface surfaces, and comparison of LLM with other coupling methods. Emphasis is given to non-matching fluid–structure meshes. We present benchmark examples that illustrate the benefits and drawbacks of competing interface treatments. Realistic application problems involving the seismic response of two existing dams are considered. These include 2D modal analyses of the Koyna gravity dam, transient-response analyses of that dam with and without reduced-order modeling, incorporation of nonlinear cavitation effects, and the 3D transient-response analysis of the Morrow Point arch dam.

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1. Introduction

The prequel [35] to this paper presents the underlying theory and analytical formulation for the first application of the method of localized Lagrange multipliers (LLM) to treat the interaction between acoustic-fluid and elastic-structure finite-element (FE) models. That material includes proof-of-concept 1D examples with known analytical solutions. No realistic benchmark application examples were discussed, since those demand coverage of modeling with computer implementation and verification aspects that would have lengthened and diluted the exposition. The present paper addresses that gap. A methodology overview is given next in the interest of self-sufficiency.

The LLM treatment introduces a *kinematic frame* at the fluid–structure interface. Two multiplier fields separately connect the frame to the fluid on one side and to the structure "wet surface" on the other. Both multiplier spaces are discretized as delta functions collocated at the fluid-interface and wet-surface structural nodes. These can be physically interpreted as interaction point forces.

The main goal of this new interface treatment is *complete decoupling* of fluid and structure models, in the sense that those can be constructed separately by different teams, or with different mesh generators. Consequently, finite-element meshes will not necessarily match over the interface. This separation streamlines preprocessing in design stages where one of the models, such as the structure, is modified (e.g. by a design team) while the other is fixed. Or, conversely, the fluid level could be changing while the structure is fixed, as in reservoir filling or pumping operations. Full decoupling also simplifies the production and use of reduced-order models.

A second key goal is to allow *processing by different programs*. For dynamic analysis by direct time integration, this is achieved by combining the LLM method with a partitioned solution procedure. The solution state is advanced separately on each program. These programs exchange information through the interface as they advance in time. The advancing sequence used here departs from the well known predictor-based staggered schemes introduced in [27]; in that, the interface state is solved *implicitly* for frame accelerations and multiplier forces. The latter are back-substituted into the fluid and structure solvers as if they were produced by an external force field. The stability analysis presented in the prequel paper shows that if the same A-stable integration

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scheme, such as the trapezoidal rule, is chosen for the fluid and structure with identical timesteps, the coupled system retains unconditional stability. One obvious generalization pertains to the use of different time-integration schemes for the fluid and structure, e.g. implicit in the structure and explicit in the fluid for resolving cavitation. In such cases, solutions may not necessarily match in time either.

While the LLM method can provide modeling flexibility, new challenges may be introduced. Even if separate non-interacting models are satisfactory as regards to stability and accuracy, the introduction of interaction can have adverse effects on the coupled response. Furthermore, if the coupled components have widely different physical characteristics (stiffness, mass, etc.), the coupled system may be scale-mismatched by orders of magnitude. This can worsen the condition number of the algebraic interface system and produce unacceptable errors, particularly under long-term cyclic loading. Accuracy monitoring requires measures to assess interface-energy conservation, whereas ill-conditioning effects may be alleviated through multiplier scaling. Error measuring is one of the practical implementation aspects omitted in the prequel paper but covered here.

To assess the LLM treatment, as well as two competing methods (Mortar and direct force–motion transfer) a realistic benchmark application class is chosen: concrete dams subject to seismic excitation. The computational models represent three physical components: structure, soil and fluid. The structure and soil are formulated as displacement-based elasticity energy equations, which are discretized as conventional solid elements. The reservoir water is modeled as a linear acoustic fluid since no significant flow develops during the response timespan of interest. The displacement potential is chosen as the response variable of the governing fluid equations. This choice has the advantage of reducing the number of degrees of freedom to one per node while automatically enforcing irrotationality.

Two actual dam–reservoir configurations are studied: the Koyna gravity dam in Maharashtra, India, and the Morrow Point arch dam in Colorado, USA. A 2D plane-strain model is used for the former and a 3D model is used for the latter. Both configurations involve the interaction of the structure, near-field soil and entrained fluid. Silent boundaries are used to truncate the fluid and soil meshes. In the gravity-dam example, the analysis optionally includes inertial cavitation. This is a highly nonlinear phenomenon whereupon the water elastic modulus drops to near zero in the cavitation volume, and re-pressurizes producing traveling closure shocks. The gravity-dam problem is used also to illustrate the analysis of coupled-system vibrations and the construction and performance of reduced-order dynamic models.

In addition to the two dam configurations, two additional benchmark problems are included. First, the problem of Chopra [7], which involves the 2D interaction of an unbounded acoustic medium with a rigid wall (with prescribed motion) is used to validate the pressure calculations and the silent boundary. Second, the Bleich and Sandler [6] 1D fluid–structure interaction (FSI) problem is used to validate the cavitation treatment.

2. Localized Lagrange multipliers

2.1. Equations of motion

Finite-element discretization of a linear acoustic fluid coupled to an elastic structure with the interface treated by the LLM method yields the following semi-discrete matrix equations of motion (EOM) in terms of displacements and interface forces as discussed in [35] (damping and silent boundaries omitted for brevity):

$$\begin{bmatrix} \mathbf{M}_S & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_F & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_S \\ \ddot{\mathbf{u}}_F \\ \ddot{\boldsymbol{\lambda}}_S \\ \ddot{\boldsymbol{\lambda}}_F \\ \ddot{\mathbf{u}}_B \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_S & \mathbf{0} & \mathbf{B}_{Sn} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_F & \mathbf{0} & \mathbf{B}_{Fn} & \mathbf{0} \\ \mathbf{B}_{Sn}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{L}_{Sn} \\ \mathbf{0} & \mathbf{B}_{Fn}^T & \mathbf{0} & \mathbf{0} & -\mathbf{L}_{Fn} \\ \mathbf{0} & \mathbf{0} & -\mathbf{L}_{Sn}^T & -\mathbf{L}_{Fn}^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_S \\ \mathbf{u}_F \\ \boldsymbol{\lambda}_S \\ \boldsymbol{\lambda}_F \\ \mathbf{u}_B \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_S \\ \mathbf{f}_F \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}. \tag{1}$$

For the structure model, $\mathbf{u}_S$ is the array of structural node displacements, whereas $\mathbf{M}_S, \mathbf{K}_S$ and $\mathbf{f}_S$ denote the master mass matrix, stiffness matrix and applied-force vector, respectively, associated with $\mathbf{u}_S$. For the fluid model, $\mathbf{u}_F$ is the array of fluid node displacements, whereas $\mathbf{M}_F, \mathbf{K}_F$ and $\mathbf{f}_F$ denote the master mass matrix, stiffness matrix and applied-force vector, respectively, associated with $\mathbf{u}_F$. Over the LLM-treated FSI interface, $\mathbf{u}_B$ is the array of frame-node displacements, $\boldsymbol{\lambda}_S$ is the array of frame-to-structure interaction forces at wet structural nodes, $\boldsymbol{\lambda}_F$ is the array of frame-to-fluid interaction forces at fluid nodes, $\mathbf{B}_{Sn}$ and $\mathbf{B}_{Fn}$ are Boolean matrices that map $\boldsymbol{\lambda}_S$ and $\boldsymbol{\lambda}_F$ onto the full set of structure and fluid node forces, respectively, $\mathbf{L}_{Sn}$ and $\mathbf{L}_{Fn}$ are matrices that map frame displacements $\mathbf{u}_B$ to structure node freedoms and fluid node freedoms, respectively. Structure and fluid nodes need not coincide over the interface. A superposed dot denotes differentiation with respect to time, t.

Fluid irrotationality is enforced by the transformation $\mathbf{u}_F = \mathbf{D}_F\boldsymbol{\psi}$, where $\boldsymbol{\psi}$ collects displacement potential degrees of freedom at fluid mesh nodes; $\mathbf{D}_F$ is a generally rectangular transformation matrix. (Since the displacement potential is a scalar field, there is only one ψ freedom per node.) A congruential transformation on fluid freedoms yields

$$\begin{bmatrix} \mathbf{M}_S & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{F\psi} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_S \\ \ddot{\boldsymbol{\psi}} \\ \ddot{\boldsymbol{\lambda}}_S \\ \ddot{\boldsymbol{\lambda}}_F \\ \ddot{\mathbf{u}}_B \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_S & \mathbf{0} & \mathbf{B}_{Sn} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{F\psi} & \mathbf{0} & \mathbf{B}_{F\psi n} & \mathbf{0} \\ \mathbf{B}_{Sn}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{L}_{Sn} \\ \mathbf{0} & \mathbf{B}_{F\psi n}^T & \mathbf{0} & \mathbf{0} & -\mathbf{L}_{Fn} \\ \mathbf{0} & \mathbf{0} & -\mathbf{L}_{Sn}^T & -\mathbf{L}_{Fn}^T & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_S \\ \boldsymbol{\psi} \\ \boldsymbol{\lambda}_S \\ \boldsymbol{\lambda}_F \\ \mathbf{u}_B \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_S \\ \mathbf{f}_{F\psi} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{Bmatrix}, \tag{2}$$

in which $\mathbf{M}_{F\psi} = \mathbf{D}_F^T\mathbf{M}_F\mathbf{D}_F, \mathbf{K}_{F\psi} = \mathbf{D}_F^T\mathbf{K}_F\mathbf{D}_F, \mathbf{B}_{F\psi n} = \mathbf{D}_F^T\mathbf{B}_{Fn}$ and $\mathbf{f}_{F\psi} = \mathbf{D}_F^T\mathbf{f}_F$, where this equation corresponds to Eq. (22) in [35].

2.2. Reduced-order modeling

The reduced-order model (ROM) formulation considered here reduces the number of normal coordinates by truncating modes of the *uncoupled* problems. The resultant free-vibration eigenproblems, both of generalized symmetric type, are

$$\mathbf{K}_S\mathbf{v}_{Si} = \omega_{Si}^2\mathbf{M}_S\mathbf{v}_{Si},\ (i = 1, \ldots, m_S), \quad \mathbf{K}_{F\psi}\mathbf{v}_{Fi} = \omega_{Fi}^2\mathbf{M}_{F\psi}\mathbf{v}_{Fi},\ (i = 1, \ldots, m_F). \tag{3}$$

(The left system in (3) is sometimes called the *dry-structure* eigenproblem.) The eigenvectors are mass orthonormalized. The retained "dry" eigenvectors for the structure are $\mathbf{v}_{Si}, (i = 1, \ldots, k_S, k_S < m_S)$, which are collected as columns of matrix $\boldsymbol{\Phi}_{Sr}$. The retained eigenvectors for the fluid are $\mathbf{v}_{Fi}, (i = 1, \ldots, k_F, k_F < m_F)$, which are