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Features of a blue-sky transition in an autonomous convective flow

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ABSTRACT

The nonlinear dynamics of a buoyant air flow in the gap formed by a horizontal cylindrical heat source and a concentric isothermally cooled square enclosure have been recently studied, in the limits of a two-dimensional approximation, for different values of the system aspect ratio, defined as the ratio between the cavity side length and the minimum gap width. Accurate analyses have revealed that, for low values of the aspect ratio, the dominant pattern is the swaying motion of a buoyant plume, which undergoes a period-doubling cascade for increasing values of the Rayleigh number. On the other hand, a substantially different behaviour has been preliminarily observed for higher values of the aspect ratio, since, in a thinner gap, the flow exhibits Rayleigh-Bénard-like cellular patterns above the cylinder.

The aim of this communication is to describe the peculiar route to chaos which has been detected for this second type of geometry. The scenario consists in the progressive winding of a periodic limit cycle on an attractor whose topology resembles that of a French horn, which is a typical signature of the codimension-2 Shilnikov bifurcation from periodic dynamics to chaos. A similar feature, known as the blue-sky catastrophe, has never been reported so far for confined convection of pure fluids.

1. Introduction

Confined buoyant flows represent one of the most interesting class of infinite dimension physical systems that, depending on the thermal boundary conditions and even for simple geometry of the confinement, have been shown to express a huge variety of complex low order dynamical behaviours. In fact, since the formalisation of the Lorenz model describing for the first time a chaotic behaviour [1], confined buoyant flows have represented one of the fundamental field of study for non-linear dynamics. Many works have been carried out on the non-linear dynamics of thermal convection in basic confined configurations, such as rectangular enclosures heated from below and from the side [2,3], toroidal, rectangular or slender natural circulation loops exhibiting Lorenz-like chaotic dynamics, and heating sources inside a cavity.

In particular, the studies on the latter type of systems encompass a variety of cases of horizontal annuli formed by external envelopes, usualy circular, square or rectangular, containing concentric or eccentric heated cylinders, of various cross sections [4], or, in the asymptotic case, a line heat source [5]. For this class of buoyant flows, theoretical and experimental studies on flow transitions, including 3D numerical simulations and stability analyses, were summarized in Ref. [6]. Moreover, outstanding experimental investigations were carried out for the case of coaxial cylinders [7,8], for which an analytical description of the steady-state has been also assessed [9,10].

The present study is concerned with the infinite horizontal cavity formed between a isothermally cooled square parallelepiped and a coaxial isothermally heated cylinder, whose cross section is sketched in Fig. 1. The assumed thermal boundary conditions determine a buoyant flow that can be investigated with respect to the governing parameters of the non-dimensional description of the problem: the ratio between the cavity side length L and the minimum enclosure to cylinder gap width H, $A = L/H$, the Rayleigh number Ra and the Prandtl number Pr. With respect to this geometry, most of the works have mainly investigated thermofluid dynamic patterns and global heat transfer rates under steady state conditions [11–14].

From the standpoint of thermofluids, such a system is particularly interesting. As soon as a temperature difference is imposed between the cylinder and the enclosure, fluid motion ensues immediately in the vicinity of the horizontal mid-plane, where the tangents to the cylindrical walls are vertical. At the same time, the fluid on top of the enclosure is subject to an unstable vertical gradient, as in the Rayleigh-Bénard problem, while vertical boundary layers are formed at the

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List of symbols

A	aspect ratio
D	cylinder diameter [m]
f	dimensionless frequency
g	gravitational acceleration [m/s^2]
$\hat{\mathbf{g}}$	gravity unit vector
H	vertical gap [m]
L	cavity side length [m]
p	dimensionless pressure [Pa]
Pr	Prandtl number
Ra	Rayleigh number
t	dimensionless time
T	dimensionless temperature
$\mathbf{u}$	dimensionless velocity vector
U	velocity [m/s]
x	horizontal coordinate
y	vertical coordinate

Greek symbols

α	thermal diffusivity [m^2/s]
β	thermal expansion coefficient [1/K]
θ	temperature [K]
ϵ	neighbourhood radius for RQA [−]
ν	kinematic viscosity [m^2/s]
Π	dimensionless period
τ	time [s]
ξ	*CFL* threshold
CFL	Courant number

Subscripts

ref	reference
S	source
W	wall
x	along x coordinate
y	along y coordinate

enclosure side-walls. The simultaneous action of these patterns in a single problem is able to produce a variety of flow configurations and transition phenomena. In fact, in the last years the focus has been posed on the description of transitional phenomena between low dimensional nonlinear dynamics in the system; a preliminary 2D numerical analysis of the first bifurcations as a function of the aspect ratio A was reported by Ref. [15], for $Pr = 0.7$ (representative of air at environmental conditions). Results showed that, for low A-values ($A \leq 3.\overline{3}$), the overall dynamics of transition are governed by the confined buoyant plume arising from the cylindrical source, whilst for higher A-values ($A \geq 5$) Rayleigh-Bénard-type rolls above the heat source represent the fundamental flow structures.

A deeper insight on the dynamical transitions of the thermal plume for the cavity with aspect ratio $A = 2.5$ has been discussed in Ref. [16] on the basis of the analysis of the time series simulated through a 2D approximated numerical model. In that configuration, the observed scenario for growing Rayleigh number consisted of a period doubling cascade with a window of quasiperiodic behaviour on a T_2-torus separating the monoperiodic P_1 from the biperiodic P_2-limit cycle. In

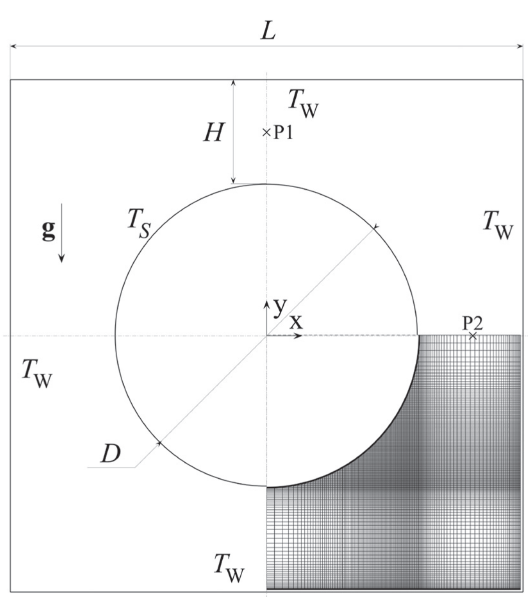

Fig. 1. Schematic view of the system under consideration and quadrant of the computational mesh.

particular, a thorough scanning of the Rayleigh number range and the accuracy of the observations, obtained with a very high level of refinement of both the geometrical grid and of the time step adopted for the simulations, allowed to report the existence of up to the P_{128}-limit cycle, i.e. of the seventh period doubling. Further results on this case were achieved with respect to the characterisation of the nature of the initial transitions, i.e. those concerning the birth and the destruction of the quasiperiodic T_2-torus separating the P_1 from the P_2 limit cycle at low Ra-values [17,18], and on the existence of two fundamental modes of the flow structure dynamics. The interactions between these two modes is responsible of the observed independent (T_2-torus) or locked states (P_n limit cycles) [19].

The present study aims at exploring the dynamical features of the flow for the case of a thinner gap, characterised by $A = 5$. On the basis of the numerical approach described in the third paragraph, a wide set of numerical simulations have been performed by refining the step of the bifurcation parameter Ra. Moreover, due to the inherently complex structure of the observed attractor and of the outstanding bifurcation, as already proposed in Ref. [20], the analysis of the transition to chaos has been addressed on the basis of the analytical tools encompassed by the Recurrence Analysis technique [21], RA in the following.

The fourth paragraph reports the analysis of the peculiar features of the simulated time series and of the topology of the attractors drawn in the state space spanned by the system variables, namely the temperature and the horizontal and vertical components of the velocity, evaluated at specified points of the domain. Aiming at the characterisation of the transitional behaviour, a thorough scanning has been performed of the range of Ra-values within which the birth of chaos occurs. This leads to the observation of general transition features resembling those related to the birth of a French horn [22,23], which is the typical attractor associated with the codimension-2 bifurcation that leads to chaos via a blue sky catastrophe. Finally, the topological characterisation is integrated and supported by the analysis of the morphological evolution of the patterns in the recurrence plots of the simulated time series, for growing Ra-values.

2. Problem statement

The problem is stated in terms of the incompressible Navier-Stokes formulation. The Oberbeck-Boussinesq approximation is enforced, all the fluid properties being consistently assumed as constant, apart from density in the buoyancy term.

The governing equations are tackled in their non-dimensional form.