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Analysis of permeabilities for slug flow in fractal porous media

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ABSTRACT

Slug flow is one of types of flow in two-phase flow in porous media, and this type of flow widely exists in oil and gas pipelines, underground water reservoirs, and nuclear reactor cooling systems, etc. Study of the mechanisms and characteristics of slug flow in porous media has the great significance in the reservoir engineering, power engineering, aerospace engineering, and chemical engineering etc. In this paper, we propose analytical models for seepage characteristics, both permeabilities and relative permeabilities, for slug flow in a capillary by unit cell approach. Then, we extend the methodology to analyze the seepage characteristics of slug flow in fractal porous media. The proposed relative permeabilities for slug flow in porous media are expressed as a function of micro-structural parameters of porous media and fluid properties, such as maximum and minimum capillary sizes, fractal dimensions, the surface tension, as well as capillary numbers. The parametrical effects on the relative permeabilities are also investigated. The validity of the proposed model for slug flow is verified by comparing the model predictions with the available experimental data.

1. Introduction

Slug flow is one of the most likely existing forms of two-phase flows, and slug flow widely exists in oil and gas pipelines, water/oil reservoirs, nuclear reactor cooling systems, heat and mass transfer in fuel cells, etc. [1–4]. Study of the mechanisms and characteristics of slug flow in porous media will help us to understand the flow behaviors in power engineering, aerospace engineering, chemical engineering and petroleum engineering etc.

Many researchers carried out a lot of theoretical and experimental investigations on flow characteristics of slug flow in single pipe [5–10]. Bretherton [11] studied the motion of infinitely long bubbles in a circular horizontal capillary with a matched-asymptotic method. Nickin et al. [12] proposed a drift flux model for slug flow in a vertical capillary. Later, Bendisken [13] investigated the motion of long bubbles in inclined tubes by experiments. He found that the Nickin's model agreed well with the experimental data in all inclination angles. Ratulowski and Chang [14] studied the flow characteristics of slug flow in ordered porous media with simple and regular microstructures by using analytical and numerical methods. Stark and Manga [15] investigated the motion of discrete bubbles in 'realistic' porous media, which were modeled as a planar network comprised of straight capillaries with different radii, by numerical method based on the extension of slug flow in single straight tube. Chen [16] extended the Bretherton's model to long and homogeneous bubbles and obtained the expressions for the relative permeabilities for steady-state slug flow of liquid and gas phases in a straight capillary tube. His model is related to the structural parameters of bubbles and viscosities of liquid and gas phases.

However, the flow characters of slug flow in irregular/disordered porous media were rarely studied. The reason may be that the microstructure of porous media is extremely complicated, and the shape/size of pores and the flow pathways are random. This brings great difficulty in investigating the slug flow in porous media. With the development of computational methods and experimental technologies, a large number of numerical methods were applied such as lattice gas automata, lattice Boltzmann method, and Monte Carlo method etc. [17–19], and new experimental technologies such as laser velocimetry, wavelet analysis, chaotic analysis, as well as nuclear magnetic resonance (NMR) [20–22] were used to study the single-phase and multiphase flows in porous media. However, the results of numerical simulations and experiments were usually reported in forms of graphs or correlations with one or more empirical constants, behind which flow mechanisms in such correlations were often ignored.

In 1982, Mandelbrot [23] founded the well-known fractal geometry, which is one of nonlinear sciences and is different from the Euclidean geometry, for describing the characteristics of irregular and disordered objects such as islands on earth and pores in porous media. In the past decades, fractal geometry has been widely applied in many areas in science and engineering since objects and processes in nature as well as in many engineering fields are found to have self-similar or statistically

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Table 1
A summary of fractal models for permeabilities of two-phase flows.

References	Year	Objectives of the model	Results of the study
Yu et al. [36]	2003	Establishment of annular flow model to obtain the permeability of fractal porous media.	The obtained fractal model has no empirical constant. The relative permeabilities of water-gas phases are functions of fractal dimensions of pore area, tortuosity, saturation as well as micro-structural parameters of porous media.
Liu et al. [37]	2007	Development of the permeability of annular flow in porous media with the capillary pressure included.	They found the capillary pressure has a significant effect on the seepage in unsaturated porous media at low saturation.
Wang and Yu [38]	2011	Study of the fluid flow characteristics in fractal-like tree networks for two-phase flows.	With capillary pressure, the analytical model for relative permeabilities of fractal-like tree branching network were obtained by fractal method.
Xu et al. [39]	2013	Investigation of the permeability in unsaturated porous media by assuming that all capillaries with the radii less than a critical radius are saturated, and the others with the radii larger than the critical radius are unsaturated.	The results indicate that the fractal permeability model of multiphase fluid related with geometrical parameters and saturation which have role in the multiphase flow in fractal porous media.
Guarracino et al. [40]	2014	Development of the relative permeabilities with the temporal evolution for porosity of fractal porous media.	The analytical expressions of multiphase flow models were obtained and the validly was verified by comparing well with available experimental data.
Miao et al. [41]	2014	Study of the spherical seepage in unsaturated porous media with the capillary pressure included.	The results shows that the contributions for rative permeabilities of the spherical seepage from capillary pressure can be ignored when $p_{cav}/p_m < 0.01$.
Xiao et al. [42]	2014	Development of the permeability of water and gas in proton exchange membrane fuel cells with fractal geometry methods and Monte Carlo method.	With the effect of capillary pressure and tortuosity of capillaries, the rative permeabilities were studied. They found that the phase fractal dimensions strongly depend on porosity.

self-similar fractal characteristics at different scales. For example, the fractal geometry theory has been shown to be powerful in characterization of porous media [24–25], and some investigators successfully applied the fractal geometry theory for permeabilities [26–29], spontaneous imbibitions [30,31] and thermal conductivities [32–35]. In addition, many investigators proposed some fractal models for two-phase porous media to predict the seepage characteristics in unsaturated porous media. Table 1 shows a summary of some investigations reported in literature. These studies were carried out based on annular flow or division phase flow. However, analysis of slug flow characteristics in porous media by applying the fractal geometry theory was not reported in literature to the best our knowledge.

Considering the effect of gas-liquid coupling, the purpose of the present work is to establish the theoretical models for seepage characteristics, permeability and relative permeability of slug flow in single capillary and in porous media based on the fractal geometry theory and technique. In the next section, the fractal geometry theory and technique for fractal porous media is briefly introduced.

2. Fractal geometry theory for porous media

It has been shown [24–26] that pore size distribution in natural porous media follows the fractal scaling law, and these media are often called fractal porous media. According to the fractal geometry theory for porous media, in a representative unit cell in a porous medium the cumulative number of pores/capillaries, whose sizes are greater than or equal to λ, can be described by the following fractal scaling law [26,43,44]

$$N(L \geq \lambda) = (\lambda_{\max}/\lambda)^{D_f} \tag{1}$$

where N is the number of capillaries and $\lambda_{\max}$is the maximum capillary diameter, respectively. D_f is the fractal dimension for pore space, generally, $0 < D_f < 2$ in two dimensions and $0 < D_f < 3$ in three dimensions.

If $\lambda = \lambda_{\min}$ in Eq. (1), Eq. (1) yields the total number of pores in a fractal set of pores/capillaries in a porous medium, i.e.

$$N_t(L \geq \lambda_{\min}) = (\lambda_{\max}/\lambda_{\min})^{D_f} \tag{2}$$

where N_t is the total number of pores/capillaries, and $\lambda_{\min}$ is the minimum diameter.

Since, in general, there are numerous pores in a set of fractal pores/capillaries in a porous medium, Eq. (1) can be considered as continuous and differentiable function. Therefore, differentiating Eq. (1) with respect to λ results in the number of pores/capillaries in the pore size interval of λ to λ + dλ,

$$-dN = D_f\lambda_{\max}^{D_f}\lambda^{-(D_f+1)}d\lambda \tag{3}$$

where $-dN > 0$, implying that the number of pores/capillaries decreases with the increase of sizes.

Dividing Eq. (3) by Eq. (2) gives:

$$-dN/N_t = D_f\lambda_{\min}^{D_f}\lambda^{-(D_f+1)}d\lambda = f(\lambda)d\lambda \tag{4a}$$

where $f(\lambda)$ is the probability density function for size distribution, i.e.

$$f(\lambda) = D_f\lambda_{\min}^{D_f}\lambda^{-(D_f+1)} \tag{4b}$$

and it should satisfy the following normalization conditions:

$$\int_{\lambda_{\min}}^{\lambda_{\max}} f(\lambda)d\lambda = 1 - (\lambda_{\min}/\lambda_{\max})^{D_f} = 1 \tag{5}$$

It is clear that Eq. (5) holds if and only if the following equation is satisfied [44]

$$(\lambda_{\min}/\lambda_{\max})^{D_f} \cong 0 \tag{6}$$

In general, in porous media $\lambda_{\min}/\lambda_{\max} \sim 10^{-2}$ or $< 10^{-2}$, and Eq. (6) can be regarded as a criterion whether the fractal geometry theory and technique can be used in porous media.

The relationship among the porosity ϕ, the fractal dimension D_f and the ratio $\lambda_{\min}/\lambda_{\max}$ in porous media is given by [26,44]

$$D_f = d_E - \frac{\ln\phi}{\ln(\lambda_{\min}/\lambda_{\max})} \tag{7}$$

where d_E is the Euclid dimension, and $d_E = 2$ in two dimensions and $d_E = 3$ in three dimensions.

According to Eq. (4b), the average pore size can be expressed as [44]

$$\lambda_{av} = \int_{\lambda_{\min}}^{\lambda_{\max}} \lambda f(\lambda)d\lambda = \frac{D_f\lambda_{\min}}{2(D_f-1)}\left[1-\left(\frac{\lambda_{\min}}{\lambda_{\max}}\right)^{1-D_f}\right] \tag{8}$$

Based on Eq. (3), the total pore area for the representative unit cell is given by

$$A_p = -\int_{\lambda_{\min}}^{\lambda_{\max}} \frac{\pi\lambda^2}{4}dN = \frac{\pi D_f\lambda_{\max}^2(1-\phi)}{4(2-D_f)} \tag{9}$$