International Communications in Heat and Mass Transfer 84 (2017) 41–48

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

An integral treatment for coupled heat and mass transfer by natural convection from a radiating vertical thin needle in a porous medium

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ARTICLE INFO

Available online 22 March 2017

MSC:
76R10
76D10
76E05
80A20

Keywords:
Buoyancy
Integral method
Heat transfer
Natural convection
Boundary layer thickness

ABSTRACT

In the present paper, an analytical study of the effects of radiation on the buoyancy induced heat and mass transfer by natural convection from an isothermal vertical thin needle embedded in a saturated porous medium has been done in case of linear temperature and concentration distribution. For the analysis of the problem, an integral method of Von Karman type has been used. The governing parameters for the problem under study are the buoyancy ratio (N), Lewis number (L_e) and radiation parameter (R_d). The computed results have covered a wide range of the governing parameters e.g. $-1 < N < 10$, $0 < L_e < 50$ and $1.4 \le R_d^* \le 10$. It has been concluded that the local Nusselt number decreases while the local Sherwood number increases along with $N > 0$ for increasing Lewis number. But an opposite trend is observed for $N < 0$ (i.e. for $N = -1$). The local Nusselt number decreases while the local Sherwood number increases for increasing values of the radiation parameter for $N > 0$. An opposing trend is observed for $N < 0$.

1. Introduction

The study of coupled heat and mass transfer due to buoyancy and radiation effects in saturated porous media is of considerable interest due to its energy-related engineering and geophysical applications such as thermal insulation of buildings, enhanced recovery of petroleum resources, filtration process, and groundwater pollution. One problem of special interest is when the intrusive magma is trapped in an aquifer such that the free convection in the groundwater is generated adjacent to the hot intrusion. Due to the concentration gradient between the intrusion and ground water, the mass transfer wall occurs simultaneously.

On account of the afore-mentioned facts only, Bejan and Khair [1] were the first researchers to report a systematic study of heat and mass transfer along a wall embedded in a saturated porous medium with constant temperature and concentration. For the general case of an axi-symmetric body of arbitrary shape, however, the literature is still very scanty. Only a few authors [2,3] have made investigations to report heat transfer related results.

From the fundamental perspectives, Nield [4] made the first attempt to study the stability of the convective flow in horizontal layers with imposed temperature and concentration gradients. This was then followed by Khan and Zebib [5] in the study of flow stability in vertical porous layer. Trevisan and Bejan [6–8] have also conducted a series of investigations of these effects on natural convection for various geometries. Yücel [9] investigated the problem of heat and mass transfer along vertical surface embedded in saturated porous media. Lai et al. [10] studied the heat and mass transfer by natural convection from slender bodies of revolution embedded in porous media. Lai and Kulacki [11] generalized the problem tackled by Bejan and Khair [1] and obtained the similarity solutions of the boundary layer equations adjacent to flat vertical surfaces with variable wall temperature and concentration,together with aiding buoyancies. Nakayama and Hossain [12] and Singh and Queeny [13] obtained integral solutions for problems for aiding buoyancies adjacent to vertical surfaces. Angirasa et al. [14] obtained the finite-difference solutions for natural convection with opposing buoyancy effects in a fluid saturated porous medium. Amahmid et al. [15] presented a numerical study for buoyancy layer type flows in a vertical porous enclosure induced by opposing buoyancy forces. Yih [16] studied the heat and mass characteristics of natural convection about a truncated cone embedded in a saturated porous medium. Chamkha [17] focused on the study of coupled heat and mass transfer by natural convection about a truncated cone in the presence of magnetic field and radiation effects. Bansod et al. [18] investigated the heat and mass transfer by natural convection from a vertical surface to a stratified Darcian fluid. Bansod [19] studied

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http://dx.doi.org/10.1016/j.icheatmasstransfer.2017.03.007
0735-1933/

the Darcy's model of buoyancy layer flows in a horizontal porous medium induced by combined buoyancy forces. Singh and Chandarki [20] studied the coupled heat and mass transfer by natural convection from a vertical cylinder embedded in a saturated porous medium.

The objective of the present paper is to extend the work of Lai et al. [10] by taking into consideration the radiative properties of the fluid. To analyse the problem, an integral approach of Von Karman type has been used.

Most of the problems governing various flow-fields in fluid mechanics are non-linear. So, it becomes significant to develop efficient methods to solve them. Ever since the advent of high speed computers, the numerical techniques for finding the solutions of highly non-linear differential equations have also been developing very quickly. However, it is still very difficult task to obtain the analytic approximations of these equations, despite the availability of much higher quality symbolic computation software such as MATHEMATICA, MATLAB, Maple, and NAG. The reason might be that we do not have a satisfactory analytic tool valid for finding the solutions of problems with stronger non-linearity. The perturbation techniques are also not applicable in all the cases, for they are essentially based on the existence of small/large parameters, called the perturbation quantities, in the equations governing the flow-fields. The absence of such perturbation quantities largely restricts the application of these perturbation techniques.

The integral technique of Von Karman type, on the other hand, is different from the perturbation techniques in the sense that it is applicable even for non-linear problems where the governing equations and/or boundary conditions do not contain any small/large parameters at all. This method with great freedom and precision provides highly accurate approximations to non-linear problems. On account of this reason only, Nakayama and Hossain [12], Singh and Queeny [13], Bansod [19], Singh and Chandarki [20] and Singh and Sharma [21] have successfully applied this integral approach to find analytical solutions of equations governing the combined heat and mass transfer by natural convection in porous media for various geometries.

The radiative effects have important applications in physics and engineering. The radiation heat transfer effects on different flows are very important in space technology and high temperature processes. But, a very little attention has been paid towards investigating the effects of radiation on boundary layers. The thermal radiation effects play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, magnetohydrodynamic accelerators, power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids. Recent developments in hypersonic flights, missile re-entry, rocket combustion chambers, power plants for inter-planetary flights and gas cooled thermal reactors have attracted the attention of researchers towards radiation as a mode of energy transfer. It is worth mentioning that unlike convection and conduction, the radiative heat transfer mechanism is rather more complex. However some reasonable approximations have been found satisfactory to make the radiative system solvable. The works of Sparrow and Cess [22], Howell [23], and Vyas and Rai [24] describe the essentials of the radiative heat transfer. Many other pertinent radiative heat transfer studies for different configurations have also been reported by researchers like Plumb et al. [25], Hossain and Takhar [26], Raptis [27], Sadeek and Salem [28], Al-Odat [29], Prasad et al. [30], Mukopadhyay [31], Vyas and Shrivastava [32], Vyas and Ranjan [33], Chauhan and Kumar [34], Baoku et al. [35], Babu et al. [36], etc. Grosan and Pop [37] studied the problem of forced convection boundary flow past nonisothermal thin needles in a nanofluid by obtaining numerical solution using the boundary value problem solver bvp4c from MATLAB. Ahmad et al. [38] investigated the problem of mixed convection boundary layer flow along vertical moving thin needles with variable heat flux using the Keller-box method. Ahmad et al. [39]also described the problem of mixed convection boundary layer flow along vertical thin needles in assisting and opposing flows with the help of Keller-box method.

2. Problem formulation

Let us consider the problem of the radiation effect on the buoyancy induced heat and mass transfer of optically dense viscous incompressible fluid by natural convection over an isothermal slender body of revolution embedded in a saturated porous medium, with a prescribed axial symmetric wall temperature T_w which is higher than the ambient temperature T_∞ (see Fig. 1). Thus, as a result of the buoyancy force, an upward convective fluid movement is induced. The variations of the fluid properties are limited to density variation which affects the buoyancy force term only. The origin of the co-ordinate system is placed at the vertex of the slender body of revolution where x represents the distance along the body of revolution and r represents the distance normal to the surface of the revolving slender body.

Here the convective fluid and porous medium have been assumed in local thermodynamic equilibrium. Further the temperature of the fluid is everywhere below its boiling point. So, the governing equations for the problem under consideration with boundary layer and Boussinesq approximations, Darcy's law and radiative fluid properties are given in cylindrical co-ordinate system as

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(ru) = 0 \tag{1}$$

$$\frac{1}{r}\frac{\partial \psi}{\partial r} = \frac{Kg}{\nu}[\beta_T(T - T_\infty) + \beta_c(c - c_\infty)] \tag{2}$$

$$\frac{\partial \psi}{\partial r}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial r} = \alpha\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{1}{\rho c_p}\frac{\partial}{\partial r}(rq_r) \tag{3}$$

$$\frac{\partial \psi}{\partial r}\frac{\partial c}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial c}{\partial r} = D\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right) \tag{4}$$

$$\rho = \rho_\infty[1 - \beta_T(T - T_\infty) - \beta_c(c - c_\infty)] \tag{5}$$

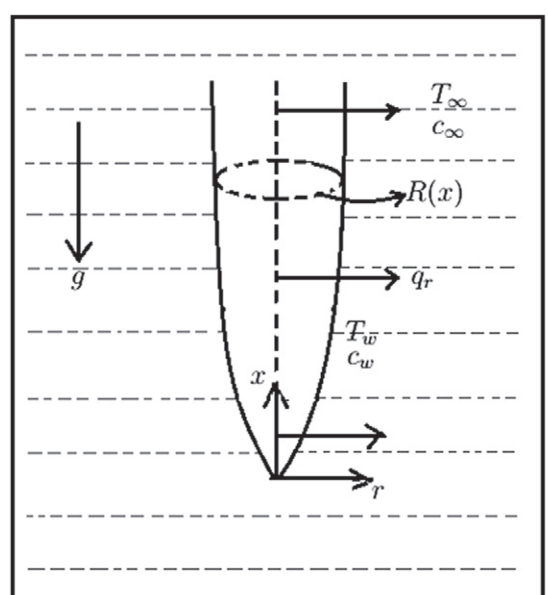

Fig. 1. A slender body of revolution embedded in saturated porous medium.