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An optimized and accurate Monte Carlo method to simulate 3D complex radiative enclosures

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ABSTRACT

The modeling of radiative heat transfer in complex radiant enclosures is a particularly challenging subject. This simulation is often best treated by calculating distribution factors through the Monte Carlo method. In order to enhance performance of the Monte Carlo method, efficient algorithms to find location of emission and direction of emission in the original Monte Carlo method are implemented. Next, the best ray tracing algorithm is introduced by comparing timing results of the USD, the BSP, the Simplex and the VVA acceleration ray tracing algorithms to make it numerically efficient as possible. Also, the constrained maximum likelihood estimation is used to enhance accuracy of the Monte Carlo by smoothing inherent random errors in the estimated distribution factors to simultaneously satisfy both of the reciprocity and summation rules. Accuracy of the Monte Carlo method is tested for a classical problem, namely a 3D box, with diffuse gray walls. For efficiency study, the optimized Monte Carlo method is then tested for two real radiative enclosures with convex and concave geometries. All ray tracing algorithms are found to result in computational gains, with respect to direct calculations that do not employ any acceleration technique. In the considered test cases, the VVA and the USD algorithms are found to be clearly superior to the BSP and the Simplex algorithms, particularly for concave geometries that have some obstructions within the computational domain.

1. Introduction

In many radiative heat transfer applications, such as radiant enclosures, geometries are very complex. Furthermore, surface properties are strongly varying both spectrally and directionally and media properties often are a function of wavelength. The versatility of the Monte Carlo method has allowed the modeling of such complex cases. In the Monte Carlo method, results are obtained by tracing a large number of rays, usually exceeding several million or even billions to achieve reasonable statistical accuracy. Thus, efficient and accurate Monte Carlo method is a prerequisite to the success of modeling of the practical applications with high levels of complexities.

The Monte Carlo method consists of several algorithms in the simulation of the radiation process such as random number generator, algorithms for finding location and direction of the emission and the ray tracing algorithm. In order to enhance performance of the Monte Carlo method, each algorithm must be optimized separately to reduce computational cost while attempting to increase efficiency of the Monte Carlo method as well.

In recent years, diverse applications of the Monte Carlo method have been reported in the literature.

Wang [1] developed an accurate stochastic algorithm for solar energy application to estimate view factors between canyon facets in the presence of shade trees and considered the potential of shade trees in mitigating canyon surface temperatures as well as saving of building energy use. In the other work, the Monte-Carlo integral method was utilized by Zhou and Qiu [2] for the direct exchange area calculation in the zone method for the modeling and simulation of the radiation transfer in an industrial furnace. The Monte Carlo method was developed by Yi et al. [3] for solving transient radiative transfer in one-dimensional scattering media with arbitrary distributions of refractive index exposed to a collimated short pulse-laser irradiation at one of its boundaries in which time shift and superposition principle was applied. Also, Kovtanyuk et al. [4] applied Monte Carlo method in the coupled radiative–conductive heat transfer mode in a chamber by two specularly and diffusely reflecting boundaries with anisotropic scattering medium. In this case a recursive Monte Carlo method was proposed and then the diffusion approximation of the radiative transfer equation was utilized to solve radiative heat transfer equation and an equation of the conductive heat exchange.

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The central computational issue in the Monte Carlo method is the determination of the intersection point between an emitted ray and the boundary faces. Several well-known search algorithms such as Octree [5], binary spatial partitioning (BSP) [6] and uniform spatial division (USD) [7] are available to narrow down the search such that all boundary faces are not searched for each ray. The USD algorithm has been used by Zeeb [8] for Monte Carlo calculations in nonparticipating media in complex geometries containing between 1000 and 5000 surfaces. The maximum obtained speedup ratio in this study was as great as 81. The proposed ray tracing algorithm was found to be 33% to 45% faster than the earlier version of ray-plane intersection which focused on the point of intersection instead of calculating the intersection distance. Mazumder [9] examined BSP and volume-by-volume advancement (VVA) algorithms for two classical problems, namely an open box, and a box in a box, in both 2D and 3D enclosures. The VVA algorithm obeyed the scaling law as $M^{1/2}$ whereas the CPU time of the BSP method scaled super-linearly. The efficiency of the VVA algorithm was found to be superior in comparison with the BSP algorithm, especially when obstructions were placed in the geometry. The maximum computational gain in case of using VVA algorithm was a factor of 334 whereas the computational gain using the BSP algorithm was found to be a factor of 52. In more recent work Naeimi and Kowsary [10] developed a new ray-object intersection algorithm based on the well-known Simplex method from linear programming. This algorithm was examined for two benchmark problems, namely two parallel plates with gray specular surfaces and a box with gray diffuse walls both in 3D case. The advantage of this ray-object intersection method was that, it is easy to implement and by using this algorithm, number of objects which must be checked in complex geometries will be reduced considerably although the computational time of this method increases with the same order of magnitude of conventional time for simple objects.

Unfortunately, most published radiative Monte Carlo works are for simple geometries such as slabs and cubes; there are few published studies to handle arbitrary, complex geometries, however, their photon tracing algorithm has not been explained in detail. It is the purpose of the present study to give an optimized version of Monte Carlo method to enhance its performance and accuracy in radiative heat transfer simulations. Timing results are computed for two radiation furnaces with convex and concave geometries composed of tens of thousands faces to demonstrate the effects of these modifications.

2. The Monte Carlo solution method

The Monte Carlo simulation is used to determine the "*distribution factors*" as defined by Mahan [11]. To aid in understanding the issues involved, the general solution method for determining the distribution factors is presented.

2.1. Radiation distribution factors

Radiation transport between one surface or volume element to one of other surfaces or volume elements of the enclosure can be described by a radiation distribution factor, D_{ij}, which is defined as the fraction of the total radiation emitted from surface element i that is absorbed by surface element j, due to both direct radiation and all possible reflections within the enclosure. If the estimated distribution factor and the exact distribution factor are denoted by $\widehat{D}_{ij}$ and D_{ij}, respectively, they may be related as

$$D_{ij} \approx \widehat{D}_{ij} = N_{ij}/N_i. \tag{1}$$

where N_i is the total number of photons emitted by surface i and N_{ij} is the total number of photons emitted by surface i that are absorbed by surface$j$, no matter what the path. Total diffuse-specular distribution factor has the following three useful properties:

Conservation of energy

$$\sum_{j=1}^{M} D_{ij} = 1, \quad 1 \le i \le M. \tag{2}$$

Reciprocity

$$\varepsilon_i A_i D_{ij} = \varepsilon_j A_j D_{ji}, \quad 1 \le i, j \le M. \tag{3}$$

Closure

$$\sum_{i=1}^{M} \varepsilon_i A_i D_{ij} = \varepsilon_j A_j, \quad 1 \le j \le M. \tag{4}$$

In the absence of a participating medium, the net radiative heat flux on surface element i, is given as

$$q_i = \varepsilon_i \sum_{j=1}^{M} (\delta_{ij} - \widehat{D}_{ij}) \sigma T_j^4. \tag{5}$$

where M is the total number of boundary surface elements, ε_i is the hemispherical, total emissivity of surface i, δ_{ij} is the Kronecker Delta, A_j is the surface area of jth face and T_j is its temperature.

Due to the reciprocity relationship, it is obvious that almost half the information in the distribution factors matrix, $\boldsymbol{D}$, is redundant, but using this redundancy to decrease the number of estimated distribution factors, is not possible with Monte Carlo Method. Since emissions from any surface automatically calculate all distribution factors for that surface.

First law of thermodynamics in Eq. (2) is satisfied when $\widehat{D}_{ij}$ is substituted for D_{ij} since every emitted photon is ultimately absorbed. Nevertheless, because of Monte Carlo inherent statistical nature $\widehat{D}_{ij} = D_{ij}$ only as N_i approaches infinity, and as a result reciprocity rule is generally not satisfied for $\widehat{D}_{ij}$ and $\widehat{D}_{ji}$. Most enclosure geometries also involve pairs of distribution factors that should be equal due to enclosure symmetry. This condition can be enforced by specifying a set of additional equality constraints, or alternatively by inserting perfectly specularly reflecting surfaces along lines of symmetry and then solving for the distribution factors over the reduced computational domain.

2.1.1. Estimating distribution factors using constrained maximum likelihood estimation

In the context of constrained maximum likelihood estimation (CML) [12], each photon from the ith surface is treated as a Bernoulli experiment with a binary outcome which the photon is either absorbed by the jth surface, or it is not. Furthermore, the exact distribution factor, D_{ij}, is interpreted as the probability that a photon emitted from the ith surface will ultimately be absorbed by the jth surface. Let $\boldsymbol{X}_{ij}$ represent a vector containing N_i random binary variables used to store the result of each Bernoulli experiment; once the kth experiment is performed, $X_{ij}^{\,k}$ is assigned a numerical value $x_{ij}^{\,k}$, where if it is absorbed by the jth surface $x_{ij}^{\,k} = 1$; otherwise, $x_{ij}^{\,k} = 0$. The resulting probability mass function of $X_{ij}^{\,k}$ is then

$$f(x_{ij}^k) = D_{ij}^{x_{ij}^k} (1 - D_{ij})^{1 - x_{ij}^k}, \quad x_{ij}^k \in \{0, 1\}. \tag{6}$$

Furthermore, since each Bernoulli experiment is independent, the probability of the entire solution set occurring is given by

$$L_{ij}(D_{ij}) = \prod_{k=1}^{N_i} D_{ij}^{x_{ij}^k} (1 - D_{ij})^{1 - x_{ij}^k} = D_{ij}^{N_{ij}} (1 - D_{ij})^{N_i - N_{ij}}. \tag{7}$$

where $L_{ij}(D_{ij})$ is the likelihood function of D_{ij}. The value of D_{ij} is estimated by finding the value $\widehat{D}_{ij}$ that maximizes the probability of the experimentally observed results occurring. Because the monotonically increasing behavior of the natural log function, this is equivalent to finding the value of $\widehat{D}_{ij}$ that maximizes