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A closed-form solution for laminar film condensation from downward pure vapour flow in vertical tubes

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ABSTRACT

An analytical solution is derived for the film thickness for simplified steady-state governing equations of laminar film condensation from laminar pure vapours flowing downward in vertical tubes. This approach yields an accurate, approximate closed-form non-marching solution for the condensate film thickness. All other relevant quantities such as the heat transfer coefficient, the vapour and liquid velocity profiles, the vapour and liquid mass flow rates, the interfacial shear stress, and the pressure gradient can be easily computed in closed-form from this solution directly at any given axial location. The present solution compares very well to other analytical works that require more complicated iterative techniques with a marching solution approach.

1. Introduction

Condensation heat transfer is important for many heat exchanger applications, for industrial applications such as desalination, and for safety analyses in the nuclear power generation industry. Since the original work of Nusselt [1], which, after some simplifying assumptions, resulted in a closed-form solution for the condensate film thickness in laminar film condensation from a pure quiescent vapour on a vertical isothermal flat plate, a large number of experimental and theoretical studies have been carried out that contributed to the understanding of this complex process. The focus of the present work is on theoretical modelling of laminar film condensation with co-current downward laminar vapour flow in a vertical tube.

For film condensation in a vertical tube with downward flow of a vapour, the coupled heat and mass transfer, interfacial shear stress, and change in velocity of the vapour flow lead to a relatively complicated set of governing equations for heat, mass, and momentum conservation in both phases and at the liquid-vapour interface. Because of this complexity, previous theoretical approaches vary in the amount of detail in their models. Less-detailed models make simplifying assumptions regarding the flow conditions in the liquid and the vapour and the interfacial shear stress. Detailed models perform numerical solutions of the complete set of parabolic governing equations. In these theoretical analyses, computations of the axial variation of film thickness and local heat transfer coefficient have been achieved through solution schemes that involve a marching procedure along the tube and possibly iteration at each axial station.

Dobran and Thorsen [2] modeled laminar flow in both the vapour core and the liquid film and performed an integral analysis. They assumed parabolic velocity and temperature profiles across the film and a parabolic velocity profile in the core and solved numerically a set of ordinary differential equations in a marching approach along the tube. Pohner and Desai [3] assumed polynomial profiles for velocity and temperature in each phase and developed a model for a laminar film with either a turbulent or a laminar vapour core. In the case of a turbulent core, the interfacial shear stress was defined using a turbulent friction factor and the interfacial heat flux was evaluated using a modified form of the Dittus-Boelter equation. For both laminar and turbulent core flow, the solution approach was a marching integration of a set of ordinary differential equations with the solution of non-linear equations at each axial location. Chen and Ke [4] modeled a turbulent flow in the vapour with a laminar or turbulent film. Chen and Ke assumed self-similar velocity profiles in the vapour and developed a three-region eddy viscosity model. They solved ordinary differential equations by marching along the tube; iteration was needed at each axial location. Revankar and Pollock [5] used an approach similar to that in [4] for laminar flow in the film and a turbulent flow of a gas-vapour mixture. They used an empirical correlation for a friction factor to determine the interfacial shear stress and a mixing length model was applied to the mixture region. A finite difference method was used to solve the resulting governing equations in a marching method. Carey [6] used a Nusselt-type analysis starting with a momentum balance on a differential element in the liquid film. He also employed a fictitious vapour density

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Nomenclature

Ar	Archimedes number, $(g\rho_L(\rho_L-\rho_V)r_o^3/\mu_L^2)$
C_P	specific heat [J kg^{-1} K^{-1}]
F_1, F_2, F_3	functions used in Eqs. (21) and (24)
g	gravitational acceleration [m s^{-2}]
h_{fg}	latent heat of vapourisation [J kg^{-1}]
h	local heat transfer coefficient [W m^{-2} K^{-1}]
Ja	Jakob number, $(C_{P,L}\Delta T/h_{fg})$
k	thermal conductivity [W m^{-1} K^{-1}]
L	tube length [m]
$\dot{m}$	mass flow rate [kg s^{-1}]
$\dot{m}^*$	dimensionless mass flow rate $(\dot{m}/\dot{m}_{\text{in}})$
Nu	Nusselt Number
P	pressure [N m^{-2}]
Pr	Prandtl number, $(\mu C_P/k)$
r	radial coordinate [m]
r_o	radius of tube [m]
r^*	dimensionless radial coordinate (r/r_o)
R^*	dimensionless film thickness variable, $(1-\delta^*)^2$
Re_{in}	inlet Reynolds number, $(2\rho_V\overline{u}_{\text{in}}r_o/\mu_V)$
T	temperature [K]
T_L^*	dimensionless liquid temperature, $(T_L-T_{\text{wall}})/(T_{\text{sat}}-T_{\text{wall}})$
ΔT	saturation-to-wall temperature difference, $(T_{\text{sat}}-T_{\text{wall}})$ [K]
u	axial velocity [m s^{-1}]
$\overline{u}_{\text{in}}$	average inlet velocity [m s^{-1}]
u^*	dimensionless axial velocity, $(u/\overline{u}_{\text{in}})$
X	dimensionless film thickness variable, $(1-R^*)/2$
z	axial coordinate [m]
z^*	dimensionless axial coordinate, (z/r_o)

Greek symbols

α	density ratio, (ρ_V/ρ_L)
β	dynamic viscosity ratio, (μ_V/μ_L)
δ	condensate film thickness [m]
δ^*	dimensionless condensate film thickness, (δ/r_o)
Δ_{MDL}	relative difference at maximum deviation location
μ	dynamic viscosity [N s m^{-2}]
ν	kinematic viscosity [m^2 s^{-1}]
ρ	density [kg m^{-3}]
ρ_+	fictitious vapour density [kg m^{-3}]
τ	shear stress [N m^{-2}]
τ^*	dimensionless shear stress, $(\tau/(\mu_V\overline{u}_{\text{in}}/r_o))$

Subscripts

i	referring to the interface
in	referring to the inlet
L	referring to the liquid
Nu	Nusselt analysis
sat	referring to saturated conditions
V	referring to the vapour
wall	referring to the wall

in a body force term that represented the pressure gradients due to the body force, friction, and momentum change (deceleration). The deceleration pressure gradient was a simplified version of an expression based on one-dimensional separated two-phase flow that depends on the axial gradient of the quality. Carey used a friction factor to determine the interfacial shear stress. Carey's derivation yielded a quartic algebraic equation for the film thickness at any axial location that must be solved by iteration. Because of the update of the vapour mass flux required in the procedure, the calculation must start at the tube inlet and march along the tube. The method can be applied to laminar and turbulent vapour flows. Muñoz-Cobo et al. [7] used a force balance on a condensate element and the concept of modelling the axial pressure gradient with a body force term involving a fictitious mixture density. They considered radial variation of axial velocity in the film, used a mean velocity in the vapour (or vapour-gas mixture), and used a friction factor correlation to determine the interfacial shear stress. They derived a marching procedure to compute the film thickness for condensation from a turbulent flow of a mixture of a vapour and a non-condensable gas in a vertical tube. On a strict interpretation, the procedure is iterative because of the dependence of intermediate quantities on the film thickness. The calculation can be made non-iterative, however, by the judicious use of the initial estimate of the film thickness at each axial station. Ghiaasiaan et al. [8] developed a simplified two-fluid model of condensation from a gas-vapour mixture based on average velocities in both phases. The mixture could have dispersed droplets and the closure relations were taken from annular dispersed two-phase flow. The governing equations were simplified to a set of coupled ordinary differential equations which were integrated numerically along the tube. Dalkilic et al. [9] developed an iterative solution approach based on Carey's approach. They computed the heat transfer coefficient for flow of R134a and compared their model results (with and without a correction for waviness) with their experimental data for turbulent flow of vapour. They also compared with the original Nusselt analysis for a vertical flat plate. For cases using a lower inlet vapour mass flux, their model without the waviness correction over-predicted the heat transfer coefficient by between 12% and 35%. With the waviness correction the deviation was between 20% and 57%. The original Nusselt equation over-predicted the heat transfer coefficient by between 18% and 52%. For cases using a higher inlet vapour mass flux, the three models under-predicted the heat transfer coefficient by between 52.8% and 76.6% whereas other correlations from the literature were within 25% of the experimental data. Groff et al. [10] solved the full parabolic governing equations of laminar film condensation from a gas-vapour mixture inside a vertical tube. A uniform inlet velocity profile and a marching scheme were used. The details of the axial velocity and temperature in both phases plus the gas mass fraction profile were calculated at each axial station along with the film thickness and axial pressure gradient. Fundamental balances were applied implicitly at the liquid-mixture interface. This technique was also used for turbulent film condensation from a co-current turbulent gas-vapour mixture flow in a vertical tube [11].

Recently, Le et al. [12] produced the equivalent of the Nusselt analysis, except in a cylindrical coordinate system. They derived closed-form solutions for the laminar condensate film thickness and heat transfer from a pure quiescent vapour on the inside and outside of a curved vertical wall.

The present work presents a theoretical analysis of film condensation for downward flow of a pure vapour in a vertical tube. The tube wall temperature may be uniform or it may vary axially by a specified polynomial function. Some of the concepts for simplification are based on the work of Le et al. [12].

The novel aspects of the present work are: (1) it includes the solution of the axial momentum equation in the vapour core in addition to the momentum equation in the liquid film, (2) it enforces overall mass conservation at all axial stations, (3) it applies fundamental balances of mass, momentum, and energy at the liquid-vapour interface, and (4) it yields a closed form, non-iterative, non-marching solution scheme. Therefore, the local film thickness, local heat transfer coefficient, and the total heat transfer rate can be obtained at any axial station in a closed-form calculation that an engineer could easily perform on a calculator or in a spreadsheet. The calculation at any axial station is independent of other axial stations. Finally, the approximate distance to the axial location where the vapour mass has been totally condensed can be calculated *a priori* (*i.e.*, without any marching procedure).